# The complete 1-types of the theory $\mathrm{Th}(\mathbb{Q}_{\mathbb{Q}})$

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From this, it can be derived that any complete 1-type $p(x_1) = p \in S_1(T)$ is determined by which formulas of the forms $x_1 < q, \ \ x_1 = q, \ \ q < x_1, q \in \mathbb{Q}$, that it contains.

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