MODEL THEORY
HOMEWORK ASSIGNMENT I

Read Chapter 1.

PROBLEMS

1. (Ari, Oscar, Silas) Show that the following pairs of abelian groups are not elementarily equivalent.

(a) $\mathbb{Z}$ **and** $\mathbb{Q}$
(b) $\mathbb{Z}$ **and** $\mathbb{Z} \times \mathbb{Z}$

2. (Khizar, Pras, Trevor) Show that the class of simple groups is not elementary.

3. (Ari, Oscar, Silas) Show that the theory of finite groups is different from the theory of all groups.

4. (Khizar, Pras, Trevor) Exercise 1.4.3 of the text.

5. (Ari, Oscar, Silas) Let σ **be a signature with one binary relation symbol and no other symbols. Show that there are exactly** 2^κ **nonisomorphic** σ**-structures of cardinality** κ **for each infinite** κ**.**

6. (Khizar, Pras, Trevor) Let L **be a language in a finite signature. Show that there are at least** $\aleph_0$**-many distinct (closed)** L**-theories, and at most** $2^{\aleph_0}$**-many (closed) distinct** L**-theories.**

7. (Ari, Oscar, Silas) Show that the following classes of graphs are elementary.

(a) **The class of infinite graphs.**
(b) **The class of triangle-free graphs.**
(c) **The class of** 2**-colorable graphs.**

8. (Khizar, Pras, Trevor) Show that the class of connected graphs is not elementary.

9. (Ari, Oscar, Silas) Show that the class of well-ordered sets is not elementary.

10. (Khizar, Pras, Trevor) Exercise 1.4.2(b) of the text.

11. (Ari, Oscar, Silas) Show that if $\mathbb{A}$ **and** $\mathbb{B}$ **are nonisomorphic finite** L**-structures, then there is a sentence true in** $\mathbb{A}$ **that is false in** $\mathbb{B}$**.**

12. (Khizar, Pras, Trevor) Let $\kappa < \lambda < \mu$ be infinite cardinals. Give an example of a structure of cardinality μ that has a substructure of cardinality κ but no substructure of cardinality λ.