Elementary Diagrams

The elementary diagram

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure.

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently:

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently: assume that L is a language of signature $(\mathcal{C}, \mathcal{F}, \mathcal{R}, \mathbf{ar})$

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently: assume that L is a language of signature $(\mathcal{C}, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ and that $\mathbf{A}$ is a structure in this signature.

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently: assume that L is a language of signature $(\mathcal{C}, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ and that $\mathbf{A}$ is a structure in this signature. For each $a \in A$,

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently: assume that L is a language of signature $(\mathcal{C}, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ and that $\mathbf{A}$ is a structure in this signature. For each $a \in A$, introduce a 'new' constant symbol c_a.

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently: assume that L is a language of signature $(\mathcal{C}, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ and that $\mathbf{A}$ is a structure in this signature. For each $a \in A$, introduce a 'new' constant symbol c_a. Let $\mathcal{C}_A = \mathcal{C} \cup \{c_a \mid a \in A\}$.

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently: assume that L is a language of signature $(\mathcal{C}, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ and that $\mathbf{A}$ is a structure in this signature. For each $a \in A$, introduce a 'new' constant symbol c_a. Let $\mathcal{C}_A = \mathcal{C} \cup \{c_a \mid a \in A\}$. Then $\mathbf{A}_A$ will be the structure in the signature $(\mathcal{C}_A, \mathcal{F}, \mathcal{R}, \mathbf{ar})$

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently: assume that L is a language of signature $(\mathcal{C}, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ and that $\mathbf{A}$ is a structure in this signature. For each $a \in A$, introduce a 'new' constant symbol c_a. Let $\mathcal{C}_A = \mathcal{C} \cup \{c_a \mid a \in A\}$. Then $\mathbf{A}_A$ will be the structure in the signature $(\mathcal{C}_A, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ with universe A and

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently: assume that L is a language of signature $(\mathcal{C}, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ and that $\mathbf{A}$ is a structure in this signature. For each $a \in A$, introduce a 'new' constant symbol c_a. Let $\mathcal{C}_A = \mathcal{C} \cup \{c_a \mid a \in A\}$. Then $\mathbf{A}_A$ will be the structure in the signature $(\mathcal{C}_A, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ with universe A and

1. $(c_a)^{\mathbf{A}_A} = a$ for $a \in A$.

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently: assume that L is a language of signature $(\mathcal{C}, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ and that $\mathbf{A}$ is a structure in this signature. For each $a \in A$, introduce a 'new' constant symbol c_a. Let $\mathcal{C}_A = \mathcal{C} \cup \{c_a \mid a \in A\}$. Then $\mathbf{A}_A$ will be the structure in the signature $(\mathcal{C}_A, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ with universe A and

1. $(c_a)^{\mathbf{A}_A} = a$ for $a \in A$.

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently: assume that L is a language of signature $(\mathcal{C}, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ and that $\mathbf{A}$ is a structure in this signature. For each $a \in A$, introduce a 'new' constant symbol c_a. Let $\mathcal{C}_A = \mathcal{C} \cup \{c_a \mid a \in A\}$. Then $\mathbf{A}_A$ will be the structure in the signature $(\mathcal{C}_A, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ with universe A and

1. $(c_a)^{\mathbf{A}_A} = a$ for $a \in A$.
2. $c^{\mathbf{A}_A} = c^{\mathbf{A}}$ for $c \in \mathcal{C}$.

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently: assume that L is a language of signature $(\mathcal{C}, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ and that $\mathbf{A}$ is a structure in this signature. For each $a \in A$, introduce a 'new' constant symbol c_a. Let $\mathcal{C}_A = \mathcal{C} \cup \{c_a \mid a \in A\}$. Then $\mathbf{A}_A$ will be the structure in the signature $(\mathcal{C}_A, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ with universe A and

1. $(c_a)^{\mathbf{A}_A} = a$ for $a \in A$.
2. $c^{\mathbf{A}_A} = c^{\mathbf{A}}$ for $c \in \mathcal{C}$.

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently: assume that L is a language of signature $(\mathcal{C}, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ and that $\mathbf{A}$ is a structure in this signature. For each $a \in A$, introduce a 'new' constant symbol c_a. Let $\mathcal{C}_A = \mathcal{C} \cup \{c_a \mid a \in A\}$. Then $\mathbf{A}_A$ will be the structure in the signature $(\mathcal{C}_A, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ with universe A and

1. $(c_a)^{\mathbf{A}_A} = a$ for $a \in A$.
2. $c^{\mathbf{A}_A} = c^{\mathbf{A}}$ for $c \in \mathcal{C}$.
3. $F^{\mathbf{A}_A} = F^{\mathbf{A}}$ for $F \in \mathcal{F}$.

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently: assume that L is a language of signature $(\mathcal{C}, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ and that $\mathbf{A}$ is a structure in this signature. For each $a \in A$, introduce a 'new' constant symbol c_a. Let $\mathcal{C}_A = \mathcal{C} \cup \{c_a \mid a \in A\}$. Then $\mathbf{A}_A$ will be the structure in the signature $(\mathcal{C}_A, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ with universe A and

1. $(c_a)^{\mathbf{A}_A} = a$ for $a \in A$.
2. $c^{\mathbf{A}_A} = c^{\mathbf{A}}$ for $c \in \mathcal{C}$.
3. $F^{\mathbf{A}_A} = F^{\mathbf{A}}$ for $F \in \mathcal{F}$.

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently: assume that L is a language of signature $(\mathcal{C}, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ and that $\mathbf{A}$ is a structure in this signature. For each $a \in A$, introduce a 'new' constant symbol c_a. Let $\mathcal{C}_A = \mathcal{C} \cup \{c_a \mid a \in A\}$. Then $\mathbf{A}_A$ will be the structure in the signature $(\mathcal{C}_A, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ with universe A and

1. $(c_a)^{\mathbf{A}_A} = a$ for $a \in A$.
2. $c^{\mathbf{A}_A} = c^{\mathbf{A}}$ for $c \in \mathcal{C}$.
3. $F^{\mathbf{A}_A} = F^{\mathbf{A}}$ for $F \in \mathcal{F}$.
4. $R^{\mathbf{A}_A} = R^{\mathbf{A}}$ for $R \in \mathcal{F}$.

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently: assume that L is a language of signature $(\mathcal{C}, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ and that $\mathbf{A}$ is a structure in this signature. For each $a \in A$, introduce a 'new' constant symbol c_a. Let $\mathcal{C}_A = \mathcal{C} \cup \{c_a \mid a \in A\}$. Then $\mathbf{A}_A$ will be the structure in the signature $(\mathcal{C}_A, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ with universe A and

1. $(c_a)^{\mathbf{A}_A} = a$ for $a \in A$.
2. $c^{\mathbf{A}_A} = c^{\mathbf{A}}$ for $c \in \mathcal{C}$.
3. $F^{\mathbf{A}_A} = F^{\mathbf{A}}$ for $F \in \mathcal{F}$.
4. $R^{\mathbf{A}_A} = R^{\mathbf{A}}$ for $R \in \mathcal{F}$.

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently: assume that L is a language of signature $(\mathcal{C}, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ and that $\mathbf{A}$ is a structure in this signature. For each $a \in A$, introduce a 'new' constant symbol c_a. Let $\mathcal{C}_A = \mathcal{C} \cup \{c_a \mid a \in A\}$. Then $\mathbf{A}_A$ will be the structure in the signature $(\mathcal{C}_A, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ with universe A and

1. $(c_a)^{\mathbf{A}_A} = a$ for $a \in A$.
2. $c^{\mathbf{A}_A} = c^{\mathbf{A}}$ for $c \in \mathcal{C}$.
3. $F^{\mathbf{A}_A} = F^{\mathbf{A}}$ for $F \in \mathcal{F}$.
4. $R^{\mathbf{A}_A} = R^{\mathbf{A}}$ for $R \in \mathcal{F}$.

L_A is the language of $\mathbf{A}_A$.

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently: assume that L is a language of signature $(\mathcal{C}, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ and that $\mathbf{A}$ is a structure in this signature. For each $a \in A$, introduce a 'new' constant symbol c_a. Let $\mathcal{C}_A = \mathcal{C} \cup \{c_a \mid a \in A\}$. Then $\mathbf{A}_A$ will be the structure in the signature $(\mathcal{C}_A, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ with universe A and

1. $(c_a)^{\mathbf{A}_A} = a$ for $a \in A$.
2. $c^{\mathbf{A}_A} = c^{\mathbf{A}}$ for $c \in \mathcal{C}$.
3. $F^{\mathbf{A}_A} = F^{\mathbf{A}}$ for $F \in \mathcal{F}$.
4. $R^{\mathbf{A}_A} = R^{\mathbf{A}}$ for $R \in \mathcal{F}$.

L_A is the language of $\mathbf{A}_A$.

Definition.

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently: assume that L is a language of signature $(\mathcal{C}, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ and that $\mathbf{A}$ is a structure in this signature. For each $a \in A$, introduce a 'new' constant symbol c_a. Let $\mathcal{C}_A = \mathcal{C} \cup \{c_a \mid a \in A\}$. Then $\mathbf{A}_A$ will be the structure in the signature $(\mathcal{C}_A, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ with universe A and

1. $(c_a)^{\mathbf{A}_A} = a$ for $a \in A$.
2. $c^{\mathbf{A}_A} = c^{\mathbf{A}}$ for $c \in \mathcal{C}$.
3. $F^{\mathbf{A}_A} = F^{\mathbf{A}}$ for $F \in \mathcal{F}$.
4. $R^{\mathbf{A}_A} = R^{\mathbf{A}}$ for $R \in \mathcal{F}$.

L_A is the language of $\mathbf{A}_A$.

Definition. The **elementary diagram** of $\mathbf{A}$ is $\mathrm{Th}(\mathbf{A}_A)$.

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently: assume that L is a language of signature $(\mathcal{C}, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ and that $\mathbf{A}$ is a structure in this signature. For each $a \in A$, introduce a 'new' constant symbol c_a. Let $\mathcal{C}_A = \mathcal{C} \cup \{c_a \mid a \in A\}$. Then $\mathbf{A}_A$ will be the structure in the signature $(\mathcal{C}_A, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ with universe A and

1. $(c_a)^{\mathbf{A}_A} = a$ for $a \in A$.
2. $c^{\mathbf{A}_A} = c^{\mathbf{A}}$ for $c \in \mathcal{C}$.
3. $F^{\mathbf{A}_A} = F^{\mathbf{A}}$ for $F \in \mathcal{F}$.
4. $R^{\mathbf{A}_A} = R^{\mathbf{A}}$ for $R \in \mathcal{F}$.

L_A is the language of $\mathbf{A}_A$.

Definition. The **elementary diagram** of $\mathbf{A}$ is $\mathrm{Th}(\mathbf{A}_A)$.

Marker writes $\mathrm{Diag}_{\mathrm{el}}(\mathbf{A})$.

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently: assume that L is a language of signature $(\mathcal{C}, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ and that $\mathbf{A}$ is a structure in this signature. For each $a \in A$, introduce a 'new' constant symbol c_a. Let $\mathcal{C}_A = \mathcal{C} \cup \{c_a \mid a \in A\}$. Then $\mathbf{A}_A$ will be the structure in the signature $(\mathcal{C}_A, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ with universe A and

1. $(c_a)^{\mathbf{A}_A} = a$ for $a \in A$.
2. $c^{\mathbf{A}_A} = c^{\mathbf{A}}$ for $c \in \mathcal{C}$.
3. $F^{\mathbf{A}_A} = F^{\mathbf{A}}$ for $F \in \mathcal{F}$.
4. $R^{\mathbf{A}_A} = R^{\mathbf{A}}$ for $R \in \mathcal{F}$.

L_A is the language of $\mathbf{A}_A$.

Definition. The **elementary diagram** of $\mathbf{A}$ is $\mathrm{Th}(\mathbf{A}_A)$.

Marker writes $\mathrm{Diag}_{\mathrm{el}}(\mathbf{A})$. Hodges writes $\mathrm{eldiag}(\mathbf{A})$.

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently: assume that L is a language of signature $(\mathcal{C}, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ and that $\mathbf{A}$ is a structure in this signature. For each $a \in A$, introduce a 'new' constant symbol c_a. Let $\mathcal{C}_A = \mathcal{C} \cup \{c_a \mid a \in A\}$. Then $\mathbf{A}_A$ will be the structure in the signature $(\mathcal{C}_A, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ with universe A and

1. $(c_a)^{\mathbf{A}_A} = a$ for $a \in A$.
2. $c^{\mathbf{A}_A} = c^{\mathbf{A}}$ for $c \in \mathcal{C}$.
3. $F^{\mathbf{A}_A} = F^{\mathbf{A}}$ for $F \in \mathcal{F}$.
4. $R^{\mathbf{A}_A} = R^{\mathbf{A}}$ for $R \in \mathcal{F}$.

L_A is the language of $\mathbf{A}_A$.

Definition. The **elementary diagram** of $\mathbf{A}$ is $\mathrm{Th}(\mathbf{A}_A)$.

Marker writes $\mathrm{Diag}_{\mathrm{el}}(\mathbf{A})$. Hodges writes $\mathrm{eldiag}(\mathbf{A})$. Monk writes $\mathrm{Eldiag}(\mathbf{A})$.

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently: assume that L is a language of signature $(\mathcal{C}, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ and that $\mathbf{A}$ is a structure in this signature. For each $a \in A$, introduce a 'new' constant symbol c_a. Let $\mathcal{C}_A = \mathcal{C} \cup \{c_a \mid a \in A\}$. Then $\mathbf{A}_A$ will be the structure in the signature $(\mathcal{C}_A, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ with universe A and

1. $(c_a)^{\mathbf{A}_A} = a$ for $a \in A$.
2. $c^{\mathbf{A}_A} = c^{\mathbf{A}}$ for $c \in \mathcal{C}$.
3. $F^{\mathbf{A}_A} = F^{\mathbf{A}}$ for $F \in \mathcal{F}$.
4. $R^{\mathbf{A}_A} = R^{\mathbf{A}}$ for $R \in \mathcal{F}$.

L_A is the language of $\mathbf{A}_A$.

Definition. The **elementary diagram** of $\mathbf{A}$ is $\mathrm{Th}(\mathbf{A}_A)$.

Marker writes $\mathrm{Diag}_{\mathrm{el}}(\mathbf{A})$. Hodges writes $\mathrm{eldiag}(\mathbf{A})$. Monk writes $\mathrm{Eldiag}(\mathbf{A})$. Pillay writes $\mathrm{D}_c(\mathbf{A})$.

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently: assume that L is a language of signature $(\mathcal{C}, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ and that $\mathbf{A}$ is a structure in this signature. For each $a \in A$, introduce a 'new' constant symbol c_a. Let $\mathcal{C}_A = \mathcal{C} \cup \{c_a \mid a \in A\}$. Then $\mathbf{A}_A$ will be the structure in the signature $(\mathcal{C}_A, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ with universe A and

1. $(c_a)^{\mathbf{A}_A} = a$ for $a \in A$.
2. $c^{\mathbf{A}_A} = c^{\mathbf{A}}$ for $c \in \mathcal{C}$.
3. $F^{\mathbf{A}_A} = F^{\mathbf{A}}$ for $F \in \mathcal{F}$.
4. $R^{\mathbf{A}_A} = R^{\mathbf{A}}$ for $R \in \mathcal{F}$.

L_A is the language of $\mathbf{A}_A$.

Definition. The **elementary diagram** of $\mathbf{A}$ is $\text{Th}(\mathbf{A}_A)$.

Marker writes $\text{Diag}_{\text{el}}(\mathbf{A})$. Hodges writes $\text{eldiag}(\mathbf{A})$. Monk writes $\text{Eldiag}(\mathbf{A})$. Pillay writes $\text{D}_c(\mathbf{A})$. (Subscript c:

The elementary diagram

Let L be a language and let $\mathbf{A}$ be an L-structure. Let $\mathbf{A}_A$ be the expansion of $\mathbf{A}$ by constants and let L_A be the language of $\mathbf{A}_A$.

Said differently: assume that L is a language of signature $(\mathcal{C}, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ and that $\mathbf{A}$ is a structure in this signature. For each $a \in A$, introduce a 'new' constant symbol c_a. Let $\mathcal{C}_A = \mathcal{C} \cup \{c_a \mid a \in A\}$. Then $\mathbf{A}_A$ will be the structure in the signature $(\mathcal{C}_A, \mathcal{F}, \mathcal{R}, \mathbf{ar})$ with universe A and

1. $(c_a)^{\mathbf{A}_A} = a$ for $a \in A$.
2. $c^{\mathbf{A}_A} = c^{\mathbf{A}}$ for $c \in \mathcal{C}$.
3. $F^{\mathbf{A}_A} = F^{\mathbf{A}}$ for $F \in \mathcal{F}$.
4. $R^{\mathbf{A}_A} = R^{\mathbf{A}}$ for $R \in \mathcal{F}$.

L_A is the language of $\mathbf{A}_A$.

Definition. The **elementary diagram** of $\mathbf{A}$ is $\mathrm{Th}(\mathbf{A}_A)$.

Marker writes $\mathrm{Diag_{el}}(\mathbf{A})$. Hodges writes $\mathrm{eldiag}(\mathbf{A})$. Monk writes $\mathrm{Eldiag}(\mathbf{A})$. Pillay writes $\mathrm{D}_c(\mathbf{A})$. (Subscript c: the 'complete' diagram of $\mathbf{A}$.)

Observations

Observations

1. The elementary diagram of a structure is a Henkin theory.

Observations

1. The elementary diagram of a structure is a Henkin theory.

Observations

1. The elementary diagram of a structure is a Henkin theory.
2. Conversely, any Henkin theory is ‘the’ elementary diagram of its canonical model.

Observations

1. The elementary diagram of a structure is a Henkin theory.
2. Conversely, any Henkin theory is ‘the’ elementary diagram of its canonical model.

Observations

1. The elementary diagram of a structure is a Henkin theory.
2. Conversely, any Henkin theory is 'the' elementary diagram of its canonical model.
3. If $\mathbf{B}$ is a model of the elementary diagram of the L-structure $\mathbf{A}$,

Observations

1. The elementary diagram of a structure is a Henkin theory.
2. Conversely, any Henkin theory is ‘the’ elementary diagram of its canonical model.
3. If $\mathbf{B}$ is a model of the elementary diagram of the L-structure $\mathbf{A}$,

Observations

1. The elementary diagram of a structure is a Henkin theory.
2. Conversely, any Henkin theory is 'the' elementary diagram of its canonical model.
3. If $\mathbf{B}$ is a model of the elementary diagram of the L-structure $\mathbf{A}$, then $\mathbf{B}|_L$ contains an elementary submodel isomorphic to $\mathbf{A}$.

Observations

1. The elementary diagram of a structure is a Henkin theory.
2. Conversely, any Henkin theory is ‘the’ elementary diagram of its canonical model.
3. If $\mathbf{B}$ is a model of the elementary diagram of the L-structure $\mathbf{A}$, then $\mathbf{B}|_L$ contains an elementary submodel isomorphic to $\mathbf{A}$.
4. (Slight rephrasing of the previous observation) If $\mathbf{B}$ is a model of the elementary diagram of the L-structure $\mathbf{A}$,

Observations

1. The elementary diagram of a structure is a Henkin theory.

2. Conversely, any Henkin theory is ‘the’ elementary diagram of its canonical model.

3. If $\mathbf{B}$ is a model of the elementary diagram of the L-structure $\mathbf{A}$, then $\mathbf{B}|_L$ contains an elementary submodel isomorphic to $\mathbf{A}$.

4. (Slight rephrasing of the previous observation) If $\mathbf{B}$ is a model of the elementary diagram of the L-structure $\mathbf{A}$,

Observations

1. The elementary diagram of a structure is a Henkin theory.
2. Conversely, any Henkin theory is ‘the’ elementary diagram of its canonical model.
3. If $\mathbf{B}$ is a model of the elementary diagram of the L-structure $\mathbf{A}$, then $\mathbf{B}|_L$ contains an elementary submodel isomorphic to $\mathbf{A}$.
4. (Slight rephrasing of the previous observation) If $\mathbf{B}$ is a model of the elementary diagram of the L-structure $\mathbf{A}$, then the function $\varphi\colon \mathbf{A} \to \mathbf{B}|_L : a \mapsto c_a^{\mathbf{B}}$ is an elementary embedding.

Upward Löwenheim-Skolem Theorem

Upward Löwenheim-Skolem Theorem.

Upward Löwenheim-Skolem Theorem.
Let $\mathbf{A}$ be an infinite L structure.

Upward Löwenheim-Skolem Theorem

Upward Löwenheim-Skolem Theorem.
Let $\mathbf{A}$ be an infinite L structure. $\mathbf{A}$ has an elementary extension of cardinality κ for any $\kappa \geq \max\{|A|, |L|\}$.

Upward Löwenheim-Skolem Theorem

Upward Löwenheim-Skolem Theorem.
Let $\mathbf{A}$ be an infinite L structure. $\mathbf{A}$ has an elementary extension of cardinality κ for any $\kappa \geq \max\{|A|, |L|\}$.

Proof.

Upward Löwenheim-Skolem Theorem.
Let $\mathbf{A}$ be an infinite L structure. $\mathbf{A}$ has an elementary extension of cardinality κ for any $\kappa \geq \max\{|A|, |L|\}$.

Proof. Let $\Delta_{\mathbf{A}}$ be the elementary diagram of $\mathbf{A}$.

Upward Löwenheim-Skolem Theorem.
Let $\mathbf{A}$ be an infinite L structure. $\mathbf{A}$ has an elementary extension of cardinality κ for any $\kappa \geq \max\{|A|, |L|\}$.

Proof. Let $\Delta_{\mathbf{A}}$ be the elementary diagram of $\mathbf{A}$. Let $\mathcal{C}_{\text{new}} = \{c_\alpha \mid \alpha < \kappa\}$ be a set of κ 'new' constants.

Upward Löwenheim-Skolem Theorem.
Let $\mathbf{A}$ be an infinite L structure. $\mathbf{A}$ has an elementary extension of cardinality κ for any $\kappa \geq \max\{|A|, |L|\}$.

Proof. Let $\Delta_{\mathbf{A}}$ be the elementary diagram of $\mathbf{A}$. Let $\mathcal{C}_{\text{new}} = \{c_\alpha \mid \alpha < \kappa\}$ be a set of κ 'new' constants. The set $\Delta_{\mathbf{A}} \cup \{c_\alpha \neq c_\beta \mid \alpha < \beta < \kappa\}$ is finitely satisfiable

Upward Löwenheim-Skolem Theorem.
Let $\mathbf{A}$ be an infinite L structure. $\mathbf{A}$ has an elementary extension of cardinality κ for any $\kappa \geq \max\{|A|, |L|\}$.

Proof. Let $\Delta_{\mathbf{A}}$ be the elementary diagram of $\mathbf{A}$. Let $\mathcal{C}_{\text{new}} = \{c_\alpha \mid \alpha < \kappa\}$ be a set of κ 'new' constants. The set $\Delta_{\mathbf{A}} \cup \{c_\alpha \neq c_\beta \mid \alpha < \beta < \kappa\}$ is finitely satisfiable (in $\mathbf{A}_A$).

Upward Löwenheim-Skolem Theorem.
Let $\mathbf{A}$ be an infinite L structure. $\mathbf{A}$ has an elementary extension of cardinality κ for any $\kappa \geq \max\{|A|, |L|\}$.

Proof. Let $\Delta_{\mathbf{A}}$ be the elementary diagram of $\mathbf{A}$. Let $\mathcal{C}_{\text{new}} = \{c_\alpha \mid \alpha < \kappa\}$ be a set of κ 'new' constants. The set $\Delta_{\mathbf{A}} \cup \{c_\alpha \neq c_\beta \mid \alpha < \beta < \kappa\}$ is finitely satisfiable (in $\mathbf{A}_A$). Let $\mathbf{B}$ be a model of size κ of this set of sentences.

Upward Löwenheim-Skolem Theorem.
Let $\mathbf{A}$ be an infinite L structure. $\mathbf{A}$ has an elementary extension of cardinality κ for any $\kappa \geq \max\{|A|, |L|\}$.

Proof. Let $\Delta_{\mathbf{A}}$ be the elementary diagram of $\mathbf{A}$. Let $\mathcal{C}_{\text{new}} = \{c_\alpha \mid \alpha < \kappa\}$ be a set of κ 'new' constants. The set $\Delta_{\mathbf{A}} \cup \{c_\alpha \neq c_\beta \mid \alpha < \beta < \kappa\}$ is finitely satisfiable (in $\mathbf{A}_A$). Let $\mathbf{B}$ be a model of size κ of this set of sentences. (Why can we choose this size?)

Upward Löwenheim-Skolem Theorem.
Let $\mathbf{A}$ be an infinite L structure. $\mathbf{A}$ has an elementary extension of cardinality κ for any $\kappa \geq \max\{|A|, |L|\}$.

Proof. Let $\Delta_{\mathbf{A}}$ be the elementary diagram of $\mathbf{A}$. Let $\mathcal{C}_{\text{new}} = \{c_\alpha \mid \alpha < \kappa\}$ be a set of κ 'new' constants. The set $\Delta_{\mathbf{A}} \cup \{c_\alpha \neq c_\beta \mid \alpha < \beta < \kappa\}$ is finitely satisfiable (in $\mathbf{A}_A$). Let $\mathbf{B}$ be a model of size κ of this set of sentences. (Why can we choose this size?)

Upward Löwenheim-Skolem Theorem.
Let $\mathbf{A}$ be an infinite L structure. $\mathbf{A}$ has an elementary extension of cardinality κ for any $\kappa \geq \max\{|A|, |L|\}$.

Proof. Let $\Delta_{\mathbf{A}}$ be the elementary diagram of $\mathbf{A}$. Let $\mathcal{C}_{\text{new}} = \{c_\alpha \mid \alpha < \kappa\}$ be a set of κ 'new' constants. The set $\Delta_{\mathbf{A}} \cup \{c_\alpha \neq c_\beta \mid \alpha < \beta < \kappa\}$ is finitely satisfiable (in $\mathbf{A}_A$). Let $\mathbf{B}$ be a model of size κ of this set of sentences. (Why can we choose this size?) $\mathbf{B}|_{L_A}$ is a model of the elementary diagram of $\mathbf{A}$,

Upward Löwenheim-Skolem Theorem.
Let $\mathbf{A}$ be an infinite L structure. $\mathbf{A}$ has an elementary extension of cardinality κ for any $\kappa \geq \max\{|A|, |L|\}$.

Proof. Let $\Delta_{\mathbf{A}}$ be the elementary diagram of $\mathbf{A}$. Let $\mathcal{C}_{\text{new}} = \{c_\alpha \mid \alpha < \kappa\}$ be a set of κ 'new' constants. The set $\Delta_{\mathbf{A}} \cup \{c_\alpha \neq c_\beta \mid \alpha < \beta < \kappa\}$ is finitely satisfiable (in $\mathbf{A}_A$). Let $\mathbf{B}$ be a model of size κ of this set of sentences. (Why can we choose this size?) $\mathbf{B}|_{L_A}$ is a model of the elementary diagram of $\mathbf{A}$, so $\mathbf{A} \preceq \mathbf{B}|_L$.

Upward Löwenheim-Skolem Theorem.
Let $\mathbf{A}$ be an infinite L structure. $\mathbf{A}$ has an elementary extension of cardinality κ for any $\kappa \geq \max\{|A|, |L|\}$.

Proof. Let $\Delta_{\mathbf{A}}$ be the elementary diagram of $\mathbf{A}$. Let $\mathcal{C}_{\text{new}} = \{c_\alpha \mid \alpha < \kappa\}$ be a set of κ 'new' constants. The set $\Delta_{\mathbf{A}} \cup \{c_\alpha \neq c_\beta \mid \alpha < \beta < \kappa\}$ is finitely satisfiable (in $\mathbf{A}_A$). Let $\mathbf{B}$ be a model of size κ of this set of sentences. (Why can we choose this size?) $\mathbf{B}|_{L_A}$ is a model of the elementary diagram of $\mathbf{A}$, so $\mathbf{A} \preceq \mathbf{B}|_L$. $\square$

Łos-Vaught Test for completeness

Łos-Vaught Test for completeness

Łos-Vaught Test for completeness.

Łos-Vaught Test for completeness.
Assume that T is a consistent L-theory that

Łos-Vaught Test for completeness.
Assume that T is a consistent L-theory that

1. is κ-categorical for some $\kappa \geq |L|$, and

Łos-Vaught Test for completeness.
Assume that T is a consistent L-theory that

1. is κ-categorical for some $\kappa \geq |L|$, and

Łos-Vaught Test for completeness.

Assume that T is a consistent L-theory that

1. is κ-categorical for some $\kappa \geq |L|$, and
2. has no finite models.

Łos-Vaught Test for completeness.
Assume that T is a consistent L-theory that

1. is κ-categorical for some $\kappa \geq |L|$, and
2. has no finite models.

Łos-Vaught Test for completeness

Łos-Vaught Test for completeness.
Assume that T is a consistent L-theory that

1. is κ-categorical for some $\kappa \geq |L|$, and
2. has no finite models.

Then T is complete.

Łos-Vaught Test for completeness

Łos-Vaught Test for completeness.
Assume that T is a consistent L-theory that

1. is κ-categorical for some $\kappa \geq |L|$, and
2. has no finite models.

Then T is complete.

Proof.

Łos-Vaught Test for completeness.
Assume that T is a consistent L-theory that

1. is κ-categorical for some $\kappa \geq |L|$, and
2. has no finite models.

Then T is complete.

Proof. (By contradiction.)

Łos-Vaught Test for completeness.
Assume that T is a consistent L-theory that

1. is κ-categorical for some $\kappa \geq |L|$, and
2. has no finite models.

Then T is complete.

Proof. (By contradiction.) Assume that T is not complete.

Łos-Vaught Test for completeness.
Assume that T is a consistent L-theory that

1. is κ-categorical for some $\kappa \geq |L|$, and
2. has no finite models.

Then T is complete.

Proof. (By contradiction.) Assume that T is not complete. There must exist an L-sentence σ such that both $T \cup \{\sigma\}$ and $T \cup \{\neg\sigma\}$ are consistent.

Łos-Vaught Test for completeness.
Assume that T is a consistent L-theory that

1. is κ-categorical for some $\kappa \geq |L|$, and
2. has no finite models.

Then T is complete.

Proof. (By contradiction.) Assume that T is not complete. There must exist an L-sentence σ such that both $T \cup \{\sigma\}$ and $T \cup \{\neg\sigma\}$ are consistent. There exist infinite structures $\mathbf{A}$ and $\mathbf{B}$ such that

Łos-Vaught Test for completeness.
Assume that T is a consistent L-theory that

1. is κ-categorical for some $\kappa \geq |L|$, and
2. has no finite models.

Then T is complete.

Proof. (By contradiction.) Assume that T is not complete. There must exist an L-sentence σ such that both $T \cup \{\sigma\}$ and $T \cup \{\neg\sigma\}$ are consistent. There exist infinite structures $\mathbf{A}$ and $\mathbf{B}$ such that

1. $\mathbf{A} \models T \cup \{\sigma\}$,

Łos-Vaught Test for completeness.
Assume that T is a consistent L-theory that

1. is κ-categorical for some $\kappa \geq |L|$, and
2. has no finite models.

Then T is complete.

Proof. (By contradiction.) Assume that T is not complete. There must exist an L-sentence σ such that both $T \cup \{\sigma\}$ and $T \cup \{\neg\sigma\}$ are consistent. There exist infinite structures $\mathbf{A}$ and $\mathbf{B}$ such that

1. $\mathbf{A} \models T \cup \{\sigma\}$,

Łos-Vaught Test for completeness.
Assume that T is a consistent L-theory that

1. is κ-categorical for some $\kappa \geq |L|$, and
2. has no finite models.

Then T is complete.

Proof. (By contradiction.) Assume that T is not complete. There must exist an L-sentence σ such that both $T \cup \{\sigma\}$ and $T \cup \{\neg\sigma\}$ are consistent. There exist infinite structures $\mathbf{A}$ and $\mathbf{B}$ such that

1. $\mathbf{A} \models T \cup \{\sigma\}$,
2. $\mathbf{B} \models T \cup \{\neg\sigma\}$,

Łos-Vaught Test for completeness

Łos-Vaught Test for completeness.
Assume that T is a consistent L-theory that

1. is κ-categorical for some $\kappa \geq |L|$, and
2. has no finite models.

Then T is complete.

Proof. (By contradiction.) Assume that T is not complete. There must exist an L-sentence σ such that both $T \cup \{\sigma\}$ and $T \cup \{\neg\sigma\}$ are consistent. There exist infinite structures $\mathbf{A}$ and $\mathbf{B}$ such that

1. $\mathbf{A} \models T \cup \{\sigma\}$,
2. $\mathbf{B} \models T \cup \{\neg\sigma\}$,

Łos-Vaught Test for completeness.
Assume that T is a consistent L-theory that

1. is κ-categorical for some $\kappa \geq |L|$, and
2. has no finite models.

Then T is complete.

Proof. (By contradiction.) Assume that T is not complete. There must exist an L-sentence σ such that both $T \cup \{\sigma\}$ and $T \cup \{\neg\sigma\}$ are consistent. There exist infinite structures $\mathbf{A}$ and $\mathbf{B}$ such that

1. $\mathbf{A} \models T \cup \{\sigma\}$,
2. $\mathbf{B} \models T \cup \{\neg\sigma\}$, and

Łos-Vaught Test for completeness

Łos-Vaught Test for completeness.
Assume that T is a consistent L-theory that

1. is κ-categorical for some $\kappa \geq |L|$, and
2. has no finite models.

Then T is complete.

Proof. (By contradiction.) Assume that T is not complete. There must exist an L-sentence σ such that both $T \cup \{\sigma\}$ and $T \cup \{\neg\sigma\}$ are consistent. There exist infinite structures $\mathbf{A}$ and $\mathbf{B}$ such that

1. $\mathbf{A} \models T \cup \{\sigma\}$,
2. $\mathbf{B} \models T \cup \{\neg\sigma\}$, and
3. $|\mathbf{A}|, |\mathbf{B}| \leq |L|$

Łos-Vaught Test for completeness.
Assume that T is a consistent L-theory that

1. is κ-categorical for some $\kappa \geq |L|$, and
2. has no finite models.

Then T is complete.

Proof. (By contradiction.) Assume that T is not complete. There must exist an L-sentence σ such that both $T \cup \{\sigma\}$ and $T \cup \{\neg\sigma\}$ are consistent. There exist infinite structures $\mathbf{A}$ and $\mathbf{B}$ such that

1. $\mathbf{A} \models T \cup \{\sigma\}$,
2. $\mathbf{B} \models T \cup \{\neg\sigma\}$, and
3. $|\mathbf{A}|, |\mathbf{B}| \leq |L|$

Łos-Vaught Test for completeness.
Assume that T is a consistent L-theory that

1. is κ-categorical for some $\kappa \geq |L|$, and
2. has no finite models.

Then T is complete.

Proof. (By contradiction.) Assume that T is not complete. There must exist an L-sentence σ such that both $T \cup \{\sigma\}$ and $T \cup \{\neg\sigma\}$ are consistent. There exist infinite structures $\mathbf{A}$ and $\mathbf{B}$ such that

1. $\mathbf{A} \models T \cup \{\sigma\}$,
2. $\mathbf{B} \models T \cup \{\neg\sigma\}$, and
3. $|\mathbf{A}|, |\mathbf{B}| \leq |L| \leq \kappa$.

Łos-Vaught Test for completeness.
Assume that T is a consistent L-theory that

1. is κ-categorical for some $\kappa \geq |L|$, and
2. has no finite models.

Then T is complete.

Proof. (By contradiction.) Assume that T is not complete. There must exist an L-sentence σ such that both $T \cup \{\sigma\}$ and $T \cup \{\neg\sigma\}$ are consistent. There exist infinite structures $\mathbf{A}$ and $\mathbf{B}$ such that

1. $\mathbf{A} \models T \cup \{\sigma\}$,
2. $\mathbf{B} \models T \cup \{\neg\sigma\}$, and
3. $|\mathbf{A}|, |\mathbf{B}| \leq |L| \leq \kappa$.

By the Upward LS-Theorem,

Łos-Vaught Test for completeness.
Assume that T is a consistent L-theory that

1. is κ-categorical for some $\kappa \geq |L|$, and
2. has no finite models.

Then T is complete.

Proof. (By contradiction.) Assume that T is not complete. There must exist an L-sentence σ such that both $T \cup \{\sigma\}$ and $T \cup \{\neg\sigma\}$ are consistent. There exist infinite structures $\mathbf{A}$ and $\mathbf{B}$ such that

1. $\mathbf{A} \models T \cup \{\sigma\}$,
2. $\mathbf{B} \models T \cup \{\neg\sigma\}$, and
3. $|\mathbf{A}|, |\mathbf{B}| \leq |L| \leq \kappa$.

By the Upward LS-Theorem, there exist elementary extensions $\mathbf{A} \preceq \mathbf{A}'$, $\mathbf{B} \preceq \mathbf{B}'$ such that $|\mathbf{A}'| = \kappa = |\mathbf{B}'|$.

Łos-Vaught Test for completeness.
Assume that T is a consistent L-theory that

1. is κ-categorical for some $\kappa \geq |L|$, and
2. has no finite models.

Then T is complete.

Proof. (By contradiction.) Assume that T is not complete. There must exist an L-sentence σ such that both $T \cup \{\sigma\}$ and $T \cup \{\neg\sigma\}$ are consistent. There exist infinite structures $\mathbf{A}$ and $\mathbf{B}$ such that

1. $\mathbf{A} \models T \cup \{\sigma\}$,
2. $\mathbf{B} \models T \cup \{\neg\sigma\}$, and
3. $|\mathbf{A}|, |\mathbf{B}| \leq |L| \leq \kappa$.

By the Upward LS-Theorem, there exist elementary extensions $\mathbf{A} \preceq \mathbf{A}'$, $\mathbf{B} \preceq \mathbf{B}'$ such that $|\mathbf{A}'| = \kappa = |\mathbf{B}'|$. By κ-categoricity, $\mathbf{A}' \cong \mathbf{B}'$.

Łos-Vaught Test for completeness.
Assume that T is a consistent L-theory that

1. is κ-categorical for some $\kappa \geq |L|$, and
2. has no finite models.

Then T is complete.

Proof. (By contradiction.) Assume that T is not complete. There must exist an L-sentence σ such that both $T \cup \{\sigma\}$ and $T \cup \{\neg\sigma\}$ are consistent. There exist infinite structures $\mathbf{A}$ and $\mathbf{B}$ such that

1. $\mathbf{A} \models T \cup \{\sigma\}$,
2. $\mathbf{B} \models T \cup \{\neg\sigma\}$, and
3. $|\mathbf{A}|, |\mathbf{B}| \leq |L| \leq \kappa$.

By the Upward LS-Theorem, there exist elementary extensions $\mathbf{A} \preceq \mathbf{A}'$, $\mathbf{B} \preceq \mathbf{B}'$ such that $|\mathbf{A}'| = \kappa = |\mathbf{B}'|$. By κ-categoricity, $\mathbf{A}' \cong \mathbf{B}'$. But $\mathbf{A}' \models \sigma$ and $\mathbf{B}' \models \neg\sigma$,

Łos-Vaught Test for completeness.
Assume that T is a consistent L-theory that

1. is κ-categorical for some $\kappa \geq |L|$, and
2. has no finite models.

Then T is complete.

Proof. (By contradiction.) Assume that T is not complete. There must exist an L-sentence σ such that both $T \cup \{\sigma\}$ and $T \cup \{\neg\sigma\}$ are consistent. There exist infinite structures $\mathbf{A}$ and $\mathbf{B}$ such that

1. $\mathbf{A} \models T \cup \{\sigma\}$,
2. $\mathbf{B} \models T \cup \{\neg\sigma\}$, and
3. $|\mathbf{A}|, |\mathbf{B}| \leq |L| \leq \kappa$.

By the Upward LS-Theorem, there exist elementary extensions $\mathbf{A} \preceq \mathbf{A}'$, $\mathbf{B} \preceq \mathbf{B}'$ such that $|\mathbf{A}'| = \kappa = |\mathbf{B}'|$. By κ-categoricity, $\mathbf{A}' \cong \mathbf{B}'$. But $\mathbf{A}' \models \sigma$ and $\mathbf{B}' \models \neg\sigma$, so this is a contradiction.

Łos-Vaught Test for completeness.
Assume that T is a consistent L-theory that

1. is κ-categorical for some $\kappa \geq |L|$, and
2. has no finite models.

Then T is complete.

Proof. (By contradiction.) Assume that T is not complete. There must exist an L-sentence σ such that both $T \cup \{\sigma\}$ and $T \cup \{\neg\sigma\}$ are consistent. There exist infinite structures $\mathbf{A}$ and $\mathbf{B}$ such that

1. $\mathbf{A} \models T \cup \{\sigma\}$,
2. $\mathbf{B} \models T \cup \{\neg\sigma\}$, and
3. $|\mathbf{A}|, |\mathbf{B}| \leq |L| \leq \kappa$.

By the Upward LS-Theorem, there exist elementary extensions $\mathbf{A} \preceq \mathbf{A}'$, $\mathbf{B} \preceq \mathbf{B}'$ such that $|\mathbf{A}'| = \kappa = |\mathbf{B}'|$. By κ-categoricity, $\mathbf{A}' \cong \mathbf{B}'$. But $\mathbf{A}' \models \sigma$ and $\mathbf{B}' \models \neg\sigma$, so this is a contradiction. □

Applications of the Łos-Vaught Test

Applications of the Łos-Vaught Test

1. Let φ_n be the sentence $(\exists x_1) \cdots (\exists x_n)(\bigwedge_{i<j} x_i \neq x_j)$.

Applications of the Łos-Vaught Test

1. Let φ_n be the sentence $(\exists x_1)\cdots(\exists x_n)(\bigwedge_{i<j} x_i \neq x_j)$.

Applications of the Łos-Vaught Test

1. Let φ_n be the sentence $(\exists x_1)\cdots(\exists x_n)(\bigwedge_{i<j} x_i \neq x_j)$. The theory axiomatized by $\Phi = \{\varphi_n \mid n = 2, 3, \ldots\}$ is complete

Applications of the Łos-Vaught Test

1. Let φ_n be the sentence $(\exists x_1) \cdots (\exists x_n)(\bigwedge_{i<j} x_i \neq x_j)$. The theory axiomatized by $\Phi = \{\varphi_n \mid n = 2, 3, \ldots\}$ is complete (in the language of equality).

Applications of the Łos-Vaught Test

1. Let φ_n be the sentence $(\exists x_1) \cdots (\exists x_n)(\bigwedge_{i<j} x_i \neq x_j)$. The theory axiomatized by $\Phi = \{\varphi_n \mid n = 2, 3, \dots\}$ is complete (in the language of equality).
2. The theory of algebraically closed fields of characteristic zero is complete.

Applications of the Łos-Vaught Test

1. Let φ_n be the sentence $(\exists x_1) \cdots (\exists x_n)(\bigwedge_{i<j} x_i \neq x_j)$. The theory axiomatized by $\Phi = \{\varphi_n \mid n = 2, 3, \ldots\}$ is complete (in the language of equality).
2. The theory of algebraically closed fields of characteristic zero is complete.

Applications of the Łos-Vaught Test

1. Let φ_n be the sentence $(\exists x_1) \cdots (\exists x_n)(\bigwedge_{i<j} x_i \neq x_j)$. The theory axiomatized by $\Phi = \{\varphi_n \mid n = 2, 3, \ldots\}$ is complete (in the language of equality).
2. The theory of algebraically closed fields of characteristic zero is complete.
3. For a given prime p,

Applications of the Łos-Vaught Test

1. Let φ_n be the sentence $(\exists x_1)\cdots(\exists x_n)(\bigwedge_{i<j} x_i \neq x_j)$. The theory axiomatized by $\Phi = \{\varphi_n \mid n = 2, 3, \ldots\}$ is complete (in the language of equality).
2. The theory of algebraically closed fields of characteristic zero is complete.
3. For a given prime p,

Applications of the Łos-Vaught Test

1. Let φ_n be the sentence $(\exists x_1)\cdots(\exists x_n)(\bigwedge_{i<j} x_i \neq x_j)$. The theory axiomatized by $\Phi = \{\varphi_n \mid n = 2, 3, \ldots\}$ is complete (in the language of equality).
2. The theory of algebraically closed fields of characteristic zero is complete.
3. For a given prime p, the theory of algebraically closed fields of characteristic p is complete.

Applications of the Łos-Vaught Test

1. Let φ_n be the sentence $(\exists x_1) \cdots (\exists x_n)(\bigwedge_{i<j} x_i \neq x_j)$. The theory axiomatized by $\Phi = \{\varphi_n \mid n = 2, 3, \ldots\}$ is complete (in the language of equality).
2. The theory of algebraically closed fields of characteristic zero is complete.
3. For a given prime p, the theory of algebraically closed fields of characteristic p is complete.
4. For any field $\mathbb{F}$, the theory of $\mathbb{F}$-vector spaces satisfying Φ from above is complete.

Applications of the Łos-Vaught Test

1. Let φ_n be the sentence $(\exists x_1) \cdots (\exists x_n)(\bigwedge_{i<j} x_i \neq x_j)$. The theory axiomatized by $\Phi = \{\varphi_n \mid n = 2, 3, \ldots\}$ is complete (in the language of equality).
2. The theory of algebraically closed fields of characteristic zero is complete.
3. For a given prime p, the theory of algebraically closed fields of characteristic p is complete.
4. For any field $\mathbb{F}$, the theory of $\mathbb{F}$-vector spaces satisfying Φ from above is complete.

Applications of the Łos-Vaught Test

1. Let φ_n be the sentence $(\exists x_1) \cdots (\exists x_n)(\bigwedge_{i<j} x_i \neq x_j)$. The theory axiomatized by $\Phi = \{\varphi_n \mid n = 2, 3, \ldots\}$ is complete (in the language of equality).
2. The theory of algebraically closed fields of characteristic zero is complete.
3. For a given prime p, the theory of algebraically closed fields of characteristic p is complete.
4. For any field $\mathbb{F}$, the theory of $\mathbb{F}$-vector spaces satisfying Φ from above is complete.
5. The theory of dense linear orders without endpoints is complete.

Applications of the Łos-Vaught Test

1. Let φ_n be the sentence $(\exists x_1) \cdots (\exists x_n)(\bigwedge_{i<j} x_i \neq x_j)$. The theory axiomatized by $\Phi = \{\varphi_n \mid n = 2, 3, \ldots\}$ is complete (in the language of equality).
2. The theory of algebraically closed fields of characteristic zero is complete.
3. For a given prime p, the theory of algebraically closed fields of characteristic p is complete.
4. For any field $\mathbb{F}$, the theory of $\mathbb{F}$-vector spaces satisfying Φ from above is complete.
5. The theory of dense linear orders without endpoints is complete.

Applications of the Łos-Vaught Test

1. Let φ_n be the sentence $(\exists x_1) \cdots (\exists x_n)(\bigwedge_{i<j} x_i \neq x_j)$. The theory axiomatized by $\Phi = \{\varphi_n \mid n = 2, 3, \ldots\}$ is complete (in the language of equality).
2. The theory of algebraically closed fields of characteristic zero is complete.
3. For a given prime p, the theory of algebraically closed fields of characteristic p is complete.
4. For any field $\mathbb{F}$, the theory of $\mathbb{F}$-vector spaces satisfying Φ from above is complete.
5. The theory of dense linear orders without endpoints is complete.
6. The theory T in the language of one constant 0 and one unary function $S(x)$, which is axiomatized by

Applications of the Łos-Vaught Test

1. Let φ_n be the sentence $(\exists x_1) \cdots (\exists x_n)(\bigwedge_{i<j} x_i \neq x_j)$. The theory axiomatized by $\Phi = \{\varphi_n \mid n = 2, 3, \ldots\}$ is complete (in the language of equality).
2. The theory of algebraically closed fields of characteristic zero is complete.
3. For a given prime p, the theory of algebraically closed fields of characteristic p is complete.
4. For any field $\mathbb{F}$, the theory of $\mathbb{F}$-vector spaces satisfying Φ from above is complete.
5. The theory of dense linear orders without endpoints is complete.
6. The theory T in the language of one constant 0 and one unary function $S(x)$, which is axiomatized by

Applications of the Łos-Vaught Test

1. Let φ_n be the sentence $(\exists x_1)\cdots(\exists x_n)(\bigwedge_{i<j} x_i \neq x_j)$. The theory axiomatized by $\Phi = \{\varphi_n \mid n = 2, 3, \dots\}$ is complete (in the language of equality).
2. The theory of algebraically closed fields of characteristic zero is complete.
3. For a given prime p, the theory of algebraically closed fields of characteristic p is complete.
4. For any field $\mathbb{F}$, the theory of $\mathbb{F}$-vector spaces satisfying Φ from above is complete.
5. The theory of dense linear orders without endpoints is complete.
6. The theory T in the language of one constant 0 and one unary function $S(x)$, which is axiomatized by
    1. S is injective.

Applications of the Łos-Vaught Test

1. Let φ_n be the sentence $(\exists x_1) \cdots (\exists x_n)(\bigwedge_{i<j} x_i \neq x_j)$. The theory axiomatized by $\Phi = \{\varphi_n \mid n = 2, 3, \dots\}$ is complete (in the language of equality).
2. The theory of algebraically closed fields of characteristic zero is complete.
3. For a given prime p, the theory of algebraically closed fields of characteristic p is complete.
4. For any field $\mathbb{F}$, the theory of $\mathbb{F}$-vector spaces satisfying Φ from above is complete.
5. The theory of dense linear orders without endpoints is complete.
6. The theory T in the language of one constant 0 and one unary function $S(x)$, which is axiomatized by
    1. S is injective.

Applications of the Łos-Vaught Test

1. Let φ_n be the sentence $(\exists x_1) \cdots (\exists x_n)(\bigwedge_{i<j} x_i \neq x_j)$. The theory axiomatized by $\Phi = \{\varphi_n \mid n = 2, 3, \ldots\}$ is complete (in the language of equality).
2. The theory of algebraically closed fields of characteristic zero is complete.
3. For a given prime p, the theory of algebraically closed fields of characteristic p is complete.
4. For any field $\mathbb{F}$, the theory of $\mathbb{F}$-vector spaces satisfying Φ from above is complete.
5. The theory of dense linear orders without endpoints is complete.
6. The theory T in the language of one constant 0 and one unary function $S(x)$, which is axiomatized by
   1. S is injective.
   2. $0 \notin \mathrm{im}(S)$.

Applications of the Łos-Vaught Test

1. Let φ_n be the sentence $(\exists x_1) \cdots (\exists x_n)(\bigwedge_{i<j} x_i \neq x_j)$. The theory axiomatized by $\Phi = \{\varphi_n \mid n = 2, 3, \ldots\}$ is complete (in the language of equality).
2. The theory of algebraically closed fields of characteristic zero is complete.
3. For a given prime p, the theory of algebraically closed fields of characteristic p is complete.
4. For any field $\mathbb{F}$, the theory of $\mathbb{F}$-vector spaces satisfying Φ from above is complete.
5. The theory of dense linear orders without endpoints is complete.
6. The theory T in the language of one constant 0 and one unary function $S(x)$, which is axiomatized by
    1. S is injective.
    2. $0 \notin \mathrm{im}(S)$.

Applications of the Łos-Vaught Test

1. Let φ_n be the sentence $(\exists x_1) \cdots (\exists x_n)(\bigwedge_{i<j} x_i \neq x_j)$. The theory axiomatized by $\Phi = \{\varphi_n \mid n = 2, 3, \ldots\}$ is complete (in the language of equality).
2. The theory of algebraically closed fields of characteristic zero is complete.
3. For a given prime p, the theory of algebraically closed fields of characteristic p is complete.
4. For any field $\mathbb{F}$, the theory of $\mathbb{F}$-vector spaces satisfying Φ from above is complete.
5. The theory of dense linear orders without endpoints is complete.
6. The theory T in the language of one constant 0 and one unary function $S(x)$, which is axiomatized by
    1. S is injective.
    2. $0 \notin \text{im}(S)$.
    3. $x \neq 0$ implies $x \in \text{im}(S)$.

Applications of the Łos-Vaught Test

1. Let φ_n be the sentence $(\exists x_1) \cdots (\exists x_n)(\bigwedge_{i<j} x_i \neq x_j)$. The theory axiomatized by $\Phi = \{\varphi_n \mid n = 2, 3, \ldots\}$ is complete (in the language of equality).
2. The theory of algebraically closed fields of characteristic zero is complete.
3. For a given prime p, the theory of algebraically closed fields of characteristic p is complete.
4. For any field $\mathbb{F}$, the theory of $\mathbb{F}$-vector spaces satisfying Φ from above is complete.
5. The theory of dense linear orders without endpoints is complete.
6. The theory T in the language of one constant 0 and one unary function $S(x)$, which is axiomatized by
    1. S is injective.
    2. $0 \notin \text{im}(S)$.
    3. $x \neq 0$ implies $x \in \text{im}(S)$.

Applications of the Łos-Vaught Test

1. Let φ_n be the sentence $(\exists x_1)\cdots(\exists x_n)(\bigwedge_{i<j} x_i \neq x_j)$. The theory axiomatized by $\Phi = \{\varphi_n \mid n = 2, 3, \ldots\}$ is complete (in the language of equality).
2. The theory of algebraically closed fields of characteristic zero is complete.
3. For a given prime p, the theory of algebraically closed fields of characteristic p is complete.
4. For any field $\mathbb{F}$, the theory of $\mathbb{F}$-vector spaces satisfying Φ from above is complete.
5. The theory of dense linear orders without endpoints is complete.
6. The theory T in the language of one constant 0 and one unary function $S(x)$, which is axiomatized by
   1. S is injective.
   2. $0 \notin \mathrm{im}(S)$.
   3. $x \neq 0$ implies $x \in \mathrm{im}(S)$.

Exercise!

Exercise!

Give a complete first-order axiomatization for the field

Exercise!

Give a complete first-order axiomatization for the field

$$\mathbb{C} = \langle \{\textsf{complex numbers}\}; \cdot, +, -, 0, 1 \rangle,$$

Exercise!

Give a complete first-order axiomatization for the field

$$\mathbb{C} = \langle \{\textsf{complex numbers}\}; \cdot, +, -, 0, 1 \rangle,$$

and explain why your answer is correct.