Why the “Compactness” Theorem?

(Where is the topological space?)

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The open sets of the form O_σ corresponding to the finitely axiomatizable theories of the form $\mathrm{Th}(\sigma)$ form a basis for this topology.

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3. (zero-dimensionality) The space of complete theories has a basis of clopen sets, namely the set $\mathcal{B} = \{O_\sigma \mid \sigma \text{ a sentence}\}$ of clopen sets corresponding to finitely axiomatizable theories.

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