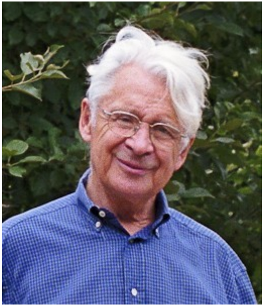
ω-categorical structures



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Simplest example. Let X be an infinite set and let $G = \text{Sym}(X)$ be the group of all permutations of the set X. G acts oligomorphically on X. (Describe the orbits of G on X, X^2, X^3, ETC.) What happens if you replace $\text{Sym}(X)$ by its subgroup $\text{Sym}_{\text{fin}}(X)$ of permutations of finite support?

Next simplest example. Let $G = \text{Aut}(V)$ where V is an ω-dimensional vector space over a finite field $\mathbb{F}$.

Oligomorphic group actions

Oligo- is a Greek prefix meaning "few".

If a group G acts on a set X, then it acts "diagonally" on X^n for any n. Diagonal action means, if $g \in G$ and $(x_1, \ldots, x_n) \in X^n$, then

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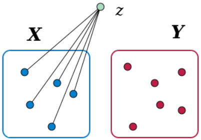

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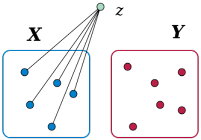

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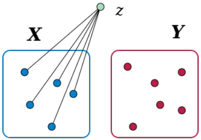

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The theory axiomatized by $\{P(m, n) \mid m, n \in \omega\}$ is complete and ω-categorical. Its unique countable model is called the **random graph**.

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