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This slogan indicates that, while algebraic geometers may study curves, surfaces, etc, that are defined over a field, a model theorist studies arbitrary definable sets over arbitrary structures.

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The first two examples are 1-sorted structures, while the third is a 2-sorted structure.

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$$(\exists \ell)(I(p, \ell) \wedge I(q, \ell) \wedge I(r, \ell))$$

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First-order model theory starts with an alphabet of symbols, and then explains how to generate and associated triple $(\mathcal{K}, \Sigma, \models)$ where $\mathcal{K}$ is the class of all first-order structures in this alphabet, Σ is a class of all first-order sentences in this alphabet, and $\models$ is a satisfaction relation.