HW 3.

1. (Bender-Stone, Green, Rodriguez) Show that the following statement is equivalent over ZFC to CH:

   The real plane $\mathbb{R}^2$ *may be partitioned into two cells,* $\{A, B\}$*, in such a way that any horizontal line in* $\mathbb{R}^2$ *has countable intersection with* A *and any vertical line in* $\mathbb{R}^2$ *has countable intersection with* B*.*

2. (Bender-Stone) Show that if κ and λ are infinite cardinals and $\kappa < \lambda$, then there is an infinite cardinal μ such that $\mu^\kappa < \mu^\lambda$.

3. (Green) Show that the following statement is equivalent over ZFC to GCH:
   If κ is any infinite cardinal, then $\kappa^{\mathrm{cf}(\kappa)} = \kappa^+$.

3. (Rodriguez) Let $\mathbb{F}$ be a field of size κ and let V be an $\mathbb{F}$-vector space of infinite dimension λ.
   (a) Show that the dimension of the dual space V^* is κ^λ.
   (b) Use the Main Theorem of Cardinal Arithmetic to simplify $\dim_{\mathbb{F}}(V^*)$ in the case where $|\mathbb{F}| = \beth_{\omega_1}$ and $\dim_{\mathbb{F}}(V) = \aleph_0$.