Practice Problems about Length, Angle, Span.

Let $\mathbf{e}_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}$, $\mathbf{e}_2 = \begin{bmatrix}0\\1\\0\end{bmatrix}$, $\mathbf{e}_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}$ be the standard basis vectors for $\mathbb{R}^3$.

(1) What is the length of $\mathbf{e}_i$?

(2) What is the angle between $\mathbf{e}_i$ and $\mathbf{e}_j$? (Two cases!)

Definition 2.13 (page 100). The **span** (or **linear closure**) of a set V of vectors is the set of all linear combinations of vectors from V. (If $S = \emptyset$, then $\mathsf{span}(V) = \{0\}$.)

(1) What is the span of $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\} \subseteq \mathbb{R}^3$?

(2) What is the span of $\{\mathbf{e}_1, \mathbf{e}_2\} \subseteq \mathbb{R}^3$?

(3) What is the span of $\{\mathbf{e}_1\} \subseteq \mathbb{R}^3$?

(4) If $V \subseteq \mathbb{R}^1$, then what are the possibilities for $\mathsf{span}(V)$?

(5) If $V \subseteq \mathbb{R}^2$, then what are the possibilities for $\mathsf{span}(V)$?

(6) If $V \subseteq \mathbb{R}^3$, then what are the possibilities for $\mathsf{span}(V)$?