Propositional logic

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- W = "The ground is wet".

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R	W	$R \wedge W$
0	0	0
0	1	0
1	0	0
1	1	1

R	W	$R \vee W$
0	0	0
0	1	1
1	0	1
1	1	1

R	$\neg R$
0	1
1	0

R	W	$R \rightarrow W$
0	0	1
0	1	1
1	0	0
1	1	1

R	W	$R \leftrightarrow W$
0	0	1
0	1	0
1	0	0
1	1	1

R	W	$R \oplus W = R \veebar W$
0	0	0
0	1	1
1	0	1
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Write the truth table for $P := R \vee (((A \wedge B) \to (\neg R)))$.

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A	B	R	$A \wedge B$	$\neg R$	$((A \wedge B) \rightarrow (\neg R))$	P
0	0	0	0	1	1	1
0	0	1	0	0	1	1
0	1	0	0	1	1	1
0	1	1	0	0	1	1
1	0	0	0	1	1	1
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This proposition P is a **tautology**, because it assumes the value “true” under any truth assignment to the propositional variables.

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This proposition P is a **tautology**, because it assumes the value "true" under any truth assignment to the propositional variables. This means that P is true because of its logical structure alone, and not because of the truth values of its variables.

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(Write $P \equiv Q$.)

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- ($\to$ is redundant) $(P \to Q) \equiv (\neg P) \vee Q$

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The truth table of a monomial has exactly one row whose value is $T = 1$:

A	B	C	D	$(\neg A) \wedge B \wedge C \wedge (\neg D)$
0	0	0	0	0
				$\vdots$
0	1	1	0	1
				$\vdots$
1	1	1	1	0

The monomial $(\neg A) \wedge B \wedge C \wedge (\neg D)$ assumes value 1 iff $A = 0, B = 1, C = 1, D = 0$.

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Small example. Create a proposition with truth table

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$$((\neg A) \wedge B \wedge (\neg C)) \vee (A \wedge (\neg B) \wedge C) \vee (A \wedge B \wedge C)$$

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Corollary. The symbols $\wedge, \vee, \neg$ are a "complete" set of logical connectives, in the sense that any proposition is logically equivalent to one expressed with $\{\wedge, \vee, \neg\}$ + propositional variables.

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If the set of propositional variables to be considered is $\{A, B, C\}$, then the DNF for A is

$$(A \wedge (\neg B) \wedge (\neg C)) \vee (A \wedge (\neg B) \wedge C) \vee (A \wedge B \wedge (\neg C)) \vee (A \wedge B \wedge C)$$

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and read this "there exists x such that $\varphi(x)$" or "there exists x such that $(x < 0)$". This is true in the real numbers, and the assertion that it is true means "there is some x in $\mathbb{R}$ such that $x < 0$". (For example, $x = -1$ is a value in $\mathbb{R}$ that winesses the truth of this statement.)

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and read this "there exists x such that $\varphi(x)$" or "there exists x such that $(x < 0)$". This is true in the real numbers, and the assertion that it is true means "there is some x in $\mathbb{R}$ such that $x < 0$". (For example, $x = -1$ is a value in $\mathbb{R}$ that winesses the truth of this statement.)

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We will describe a process to determine the truth of a sentence in a structure if the sentence is written in prenex normal form.

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Dogs bark and there is a cat x with stripes $\equiv$ there is a cat x such that (dogs bark and x has stripes)

$$B \wedge (\exists x)S(x) \equiv (\exists x)(B \wedge S(x))$$

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Variable conflict

Let's consider the problem of putting the following sentence into prenex form:

$$((\exists x)(x < 0)) \wedge ((\exists x)(x > 0)).$$

1. (First step) $(\exists x)((x < 0) \wedge (\exists x)(x > 0))$ No error!
2. (Second step) $(\exists x)(\exists x)((x < 0) \wedge (x > 0))$ Error!
3. (Alternative second step) $(\exists x)((x < 0) \wedge (x > 0))$ Error!

We need to "standardize the variables apart", first. That is, rewrite the original sentence as

$$((\exists x)(x < 0)) \wedge ((\exists y)(y > 0)),$$

in order to avoid variable conflicts. A prenex form for the sentence is

$$(\exists x)(\exists y)((x < 0) \wedge (y > 0)).$$