More applications of e-vectors

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$v_n = A^n v_0 = c \cdot \lambda_1^n \cdot w_1 + d \cdot \lambda_2^n \cdot w_2 = c \cdot w_1 + d \cdot (-7)^n \cdot w_2$. Eventually the tiger or the deer population will no longer be positive.

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The two new axes, or variables, do not interact.

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(Note that $P_{n+1} + Q_{n+1} = P_n + Q_n$, so the total population of all states is assumed to remain constant as $n \to \infty$.)

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