

## Practice with tables! (Solutions in blue!)

Let  $\mathbb{A} = \langle U, V; +, \diamond, \Box, \sqsubseteq \rangle$  be a structure where

- (1)  $U = \{a, b\}$ ,  $V = \{p, q\}$ ,
- (2)  $+: U \times V \rightarrow U$  is a binary operation from  $U$  and  $V$  to  $U$ ,
- (3)  $\diamond: U \times U \rightarrow V$  is a binary operation from  $U$  to  $V$ ,
- (4)  $\Box: U \rightarrow V$  is a unary operation from  $U$  to  $V$ ,
- (5)  $\sqsubseteq: V \times V \rightarrow \{\top, \perp\}$  is a binary predicate.

Suppose the tables for these structural elements are

$x$	$y$	$x + y$
$a$	$p$	$a$
$a$	$q$	$a$
$b$	$p$	$b$
$b$	$q$	$a$

$x$	$y$	$x \diamond y$
$a$	$a$	$p$
$a$	$b$	$q$
$b$	$a$	$q$
$b$	$b$	$q$

$x$	$\Box x$
$a$	$q$
$b$	$p$

$x$	$y$	$x \sqsubseteq y$
$p$	$p$	$\top$
$p$	$q$	$\perp$
$q$	$p$	$\perp$
$q$	$q$	$\top$

Create tables for these compound structural elements. If you have time, draw tree representations.

- (1) The compound operation  $(x \diamond (x + \Box x))$ .

$x$	$\Box x$	$x + \Box x$	$x \diamond (x + \Box x)$
$a$	$q$	$a$	$p$
$b$	$p$	$b$	$q$

- (2) The compound operation  $((x + y) \diamond (x + z))$ .

Since  $x$  is a left input for  $+$ , we must have  $x \in \{a, b\}$ . Since  $y$  and  $z$  are right inputs for  $+$ , we must have  $y, z \in \{p, q\}$ .

$x$	$y$	$z$	$x + y$	$x + z$	$(x + y) \diamond (x + z)$
$a$	$p$	$p$	$a$	$a$	$p$
$a$	$p$	$q$	$a$	$a$	$p$
$a$	$q$	$p$	$a$	$a$	$p$
$a$	$q$	$q$	$a$	$a$	$p$
$b$	$p$	$p$	$b$	$b$	$q$
$b$	$p$	$q$	$b$	$a$	$q$
$b$	$q$	$p$	$a$	$b$	$q$
$b$	$q$	$q$	$a$	$a$	$p$

- (3) The compound predicate  $\Box x \sqsubseteq (x \diamond x)$ .  
 (This could be written in prefix notation as  $\sqsubseteq (\Box(x), \diamond(x, x))$ .)

$x$	$\Box x$	$(x \diamond x)$	$\Box x \sqsubseteq (x \diamond x)$
$a$	$q$	$p$	$\perp$
$b$	$p$	$q$	$\perp$