Set Theory as a Foundation for Mathematics

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There will be 4.

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So, we can't know precisely how or when a primitive concept like the number 2 came into being, but mathematics was reorganized during the late 1800s–early 1900s (Dedekind, Cantor, Peano, Zermelo, Hilbert), and we can now know the precise definition that mathematicians have chosen for the number "2".

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I did not provide a "real definition" of "set", because I have not tried to make the concept of "set" definite. To do that, I would have to first define "unordered", "collection", "distinct", "element", which I won't do.

Since we want sets to be unordered collections of distinct elements

$$\{A, B\} = \{B, A\} = \{A, A, B\} = \{A, B, A, B, B, B, \ldots\}.$$

Numbers

Numbers

Some natural numbers:

Numbers

Some natural numbers:

$$0 := \{\ \}$$

Numbers

Some natural numbers:

$$\begin{aligned} 0 &:= \{\ \} \\ 1 &:= \{0\} \end{aligned}$$

Numbers

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Df. (Successor)

Numbers

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$$\begin{aligned} 0 &:= \{\ \} \\ 1 &:= \{0\} \\ 2 &:= \{0, 1\} \\ 3 &:= \{0, 1, 2\} \end{aligned}$$

Df. (Successor) $S(x) := x \cup \{x\}$.

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(Take $1 := S(0)$, $2 := S(1)$, to be definitions of the symbols $1, 2, \ldots$, etc.)

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Df. (Definition of Addition (by recursion))

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Df. (Definition of Addition (by recursion))

$$m + 0 := m \qquad \text{(Initial condition)}$$

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Ex. $(2 + 2 = ?)$

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$$2 + 2 \overset{Df}{=} 2 + S(1) \overset{RR}{=} S(2 + 1)$$

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