**DISCRETE MATH**
**MIDTERM**

**Name:**_____

You have 80 minutes for this exam. If you have a question, raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**.

(1) Fill in the blank with all appropriate choices (if any exist).

(a) Mathematics is founded on set theory, set theory, set theory.

(i) set theory.
(ii) set theory.
(iii) set theory.

(b) Naive set theory is inconsistent.

(i) free of any contradictions.

(ii) inconsistent.

(iii) called ZFC.

(c) The sentence

$$\forall x\, \forall y\, \exists p\, \forall z\, ((z \in p) \leftrightarrow (z = x \text{ or } z = y))$$

expresses the Axiom of Pairing.

(i) Extensionality

(ii) Power Set

(iii) Pairing

(2) (a) What does it mean to say $f$ is a function from $A$ to $B$? (Give the definition.)

A function from $A$ to $B$ is a relation from $A$ to $B$ that satisfies the function rule.

(b) Define inductive set.
A set is inductive if it contaons 0 and is closed under successor.

(c) Define the set of natural numbers.

The set of natural numbers is the intersection of all inductive sets.

(3) Give an example, if one exists, of each of the following. If no example exists, say why.

(a) A class that is not a proper class.

A class that is not a proper class is just a set. The simplest example is $\emptyset$.

(b) A partition of $\mathbb{N}$ with exactly two cells.

$\{\{0\}, \{n \in \mathbb{N} \mid n \neq 0\}\}$ is a partition of $\mathbb{N}$ into the singleton zero set and the set of nonzero natural numbers.

(c) A function between finite sets whose coimage is larger than its image.

No such function exists. If $f : A \to B$ is any function, then the induced map $\overline{f} : \mathrm{coim}(f) \to \mathrm{im}(f)$ is a bijection, so the coimage and image have the same size.

(4) Let $f$ be the function from $\{0, 1, 2\}$ to $\{a, b, c\}$ defined by $f(0) = f(1) = a$ and $f(2) = b$.

(a) Write $\mathrm{coim}(f)$ as a set.

$\{\{0, 1\}, \{2\}\}$.

(b) Write $\ker(f)$ as a set.

$\{(0, 0), (1, 1), (2, 2), (0, 1), (1, 0)\}$

(c) Write the induced map as a set.

$\{(\{0, 1\}, a), (\{2\}, b)\}$

(5) Write down the recursive definition of addition of natural numbers. Use the definition to show that $S(m) = m + 1$.

$$\begin{aligned} m + 0 &:= m && \text{(IC)} \\ m + S(n) &:= S(m + n) && \text{(RR)} \end{aligned}$$

For the second statement, for any $m$ we have

$$\begin{aligned} m + 1 &= m + S(0) && \text{Defn of number 1} \\ &= S(m + 0) && \text{(RR)} \\ &= S(m) && \text{(IC)} \end{aligned}$$