**Solution to Quiz 6.**

Define a function $f\colon \mathbb{N} \to \mathbb{N}$ by recursion by

$$\begin{aligned} f(0) &:= 1 \\ f(n+1) &:= 2 \cdot f(n) \end{aligned}$$

Use induction to prove that $f(k) = 2^k$ for all $k \in \mathbb{N}$.

**Solution.** Let $s_k$ be the statement "$f(k) = 2^k$".
First we prove $s_0$, the Base Case:

$$\begin{aligned} f(0) &= 1 && \text{(IC for definition of } f\text{)} \\ &= 2^0 && \text{(IC, exponentiation)} \end{aligned}$$

Now for the Inductive Step. Assume that $s_k$ is true ($f(k) = 2^k$). We shall prove that $s_{k+1}$ is true ($f(k+1) = 2^{k+1}$).

$$\begin{aligned} f(k+1) &= 2 \cdot f(k) && \text{(RR for the definition of } f\text{)} \\ &= 2 \cdot 2^k && \text{(IH)} \\ &= 2^k \cdot 2 && \text{(Commutative Law, multiplication)} \\ &= 2^{k+1} && \text{(RR, exponentiation)} \end{aligned}$$