

## Solutions to HW 7.

- Write the following propositions in disjunctive normal form, assuming that each proposition is a function of  $p$ ,  $q$  and  $r$ .

(i)  $p \rightarrow r$

$$((\neg p) \wedge (\neg q) \wedge (\neg r)) \vee ((\neg p) \wedge (\neg q) \wedge r) \vee ((\neg p) \wedge q \wedge (\neg r)) \vee ((\neg p) \wedge q \wedge r) \vee (p \wedge (\neg q) \wedge r) \vee (p \wedge q \wedge r)$$

(ii)  $((p \rightarrow q) \rightarrow ((\neg p) \leftrightarrow r))$ .

$$((\neg p) \wedge (\neg q) \wedge r) \vee ((\neg p) \wedge q \wedge r) \vee (p \wedge (\neg q) \wedge (\neg r)) \vee (p \wedge (\neg q) \wedge r) \vee (p \wedge q \wedge (\neg r))$$

(iii)  $q$

$$((\neg p) \wedge q \wedge (\neg r)) \vee ((\neg p) \wedge q \wedge r) \vee (p \wedge q \wedge (\neg r)) \vee (p \wedge q \wedge r)$$

- Write the following axioms of set theory as formal sentences.

(i) Extensionality.

$$(\forall A)(\forall B)((A = B) \leftrightarrow (\forall z)((z \in A) \leftrightarrow (z \in B)))$$

(ii) Pairing.

$$(\forall A)(\forall B)(\exists P)(\forall z)((z \in P) \leftrightarrow ((z = A) \vee (z = B)))$$

(iii) Power set.

$$(\forall A)(\exists P)(\forall z)((z \in P) \leftrightarrow \underline{(z \subseteq A)})$$

or

$$(\forall A)(\exists P)(\forall z)((z \in P) \leftrightarrow \underline{((\forall w)((w \in z) \rightarrow (w \in A)))})$$

- In 1959, Pete Seeger took lines from the Book of Ecclesiastes to write a song, which was made famous by the Byrds in 1965. One line is:

*To every thing there is a season, and a time for every purpose under heaven.*

Write this as a formal sentence using predicates  $S(s, t) =$  “ $s$  is the season for thing  $t$ ” and  $P(T, p) =$  “ $T$  is the time for purpose  $p$ ”.

$$((\forall t)(\exists s)S(s, t)) \wedge ((\forall p)(\exists T)P(T, p))$$