Solutions to HW 2.

1. Is the class of all 1-element sets a set? Explain.

No.
The class $Z = \{x \mid x \text{ has 1 element}\}$ is describable by a formula, hence it is a class. We now argue that it is not a set.
If instead Z were a set, then $\bigcup Z$ would also be a set. We shall argue that $\bigcup Z$ is the class of all sets, which is not a set.
Choose any set S. Since $S \in \{S\} \in Z$ we get $S \in \bigcup Z$, so Z contains every set.

2. Your friend offers a wager that, under the Kuratowski encoding, the ordered pair $(0,1)$ equals the natural number three. Should you take the wager? Explain.

Take the wager! (By offering the wager, the friend bets that the statement is true. We argue that it is false, so your friend will lose and you will win.)
$0 = \{\}, 1 = \{0\}, 2 = \{0,1\}$ and $3 = \{0,1,2\}$. Thus $(0,1) = \{\{0\},\{0,1\}\} = \{1,2\} \neq \{0,1,2\} = 3$. (We know that $\{1,2\} \neq \{0,1,2\}$ since only one of these two sets has an element equal to the empty set.)

(Here is a second solution: any ordered pair (x,y), considered as a set $\{\{x\},\{x,y\}\}$, contains one or two distinct elements. But 3 contains three distinct elements, so $(x,y) \neq 3$ for any x and y.)

3. Show that $\emptyset \times A = \emptyset$.

We must show that $\emptyset \times A$ has no elements. But this is clear, since if $(x,y) \in \emptyset \times A$, then $x \in \emptyset$, which is impossible ($\emptyset$ has no elements).