

Truth tables! (Read Section 3.1 of the book.)

You should memorize the tables for $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$.

p	$\neg p$
0	1
1	0

p	q	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \leftrightarrow q$
0	0	0	0	1	1
0	1	0	1	1	0
1	0	0	1	0	0
1	1	1	1	1	1

You should be able to create tables for compound truth functions (or “propositions”), such as

$$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r) :$$

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$p \rightarrow r$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	0	0	1
1	0	1	0	1	0	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

Please be able to explain/define the following.

- (1) Tautology.
- (2) Contradiction.
- (3) Logically equivalent propositions.
- (4) Logically independent propositions.
- (5) Contrapositive (of $p \rightarrow q$).
- (6) Converse (of $p \rightarrow q$).
- (7) Inverse (of $p \rightarrow q$).
- (8) Disjunctive normal form (and conjunctive normal form).
- (9) Complete set of connectives.

Practice!

(1) Show that the following pairs of propositions are logically equivalent.

(a) $p \vee p$ and p .

(b) $\neg(\neg p)$ and p .

(c) $p \rightarrow q$ and $(\neg p) \vee q$.

(d) $p \leftrightarrow q$ and $(p \rightarrow q) \wedge (q \rightarrow p)$.

(e) (DeMorgan's Laws)

(i) $\neg(p \wedge q)$ and $(\neg p) \vee (\neg q)$.

(ii) $\neg(p \vee q)$ and $(\neg p) \wedge (\neg q)$.

(2) Consider the following two statements:

(a) "If we tip the waiter in advance, then we will get good service."

(b) "If we do not tip the waiter in advance, then we will not get good service."

Are these statements logically equivalent? Are they logically independent? Are there any possibilities other than logical equivalence or logical independence?

(3) Is it a tautology to say "The Beatles were the best band of all time"?

For the next three problems let p be "It is raining." and let q be "The ground is wet."

(4) Give an example of a sentence involving p and q that is a tautology.

(5) Give an example of a sentence involving p and q that is a contradiction.

(6) Give examples of two different sentences involving p and q that are logically equivalent.