Practice with Inclusion/Exclusion, Stirling, and Bell numbers!

(1) How many positive integers less than 1000 are not divisible by 2, 3, 5 or 7?

(2) In a class of 20 students, how many study groups can be formed which include at least one of the three students Archibald, Beryl, or Cornelia? Assume that a study group must involve at least 2 students.

(3) How many 5-digit numbers fail to contain the sequence 01? How about 00?

(4) How many 6-digit numbers have the property that, for every k, the kth digit is different than the $(7-k)$th digit?

(5) A news organization reports that the percentage of voters who would be satisfied with each of three candidates A, B, C for President is 65%, 57%, 58% respectively. Furthermore, 28% would accept A or B, 30% would accept A or C, 27% would accept B or C, and 12% would accept any of the three. Is this fake news?

(6) If $f : k \to k$ is a bijection, then i is called a fixed point of f if $f(i) = i$. What percentage of bijections $f : k \to k$ have no fixed points? (Count the number of bijections with no fixed points, then divide by the total number of of bijections.)

(7) Explain why $S(n, 2) = 2^{n-1} - 1$ if $n > 0$.

(8) Explain why $S(n, n-1) = \binom{n}{2}$.

(9) Determine how the numbers $2^{n-1}, B_n, n!, 2^{n^2}$ are related to each other as n grows. (Which is larger than which?)