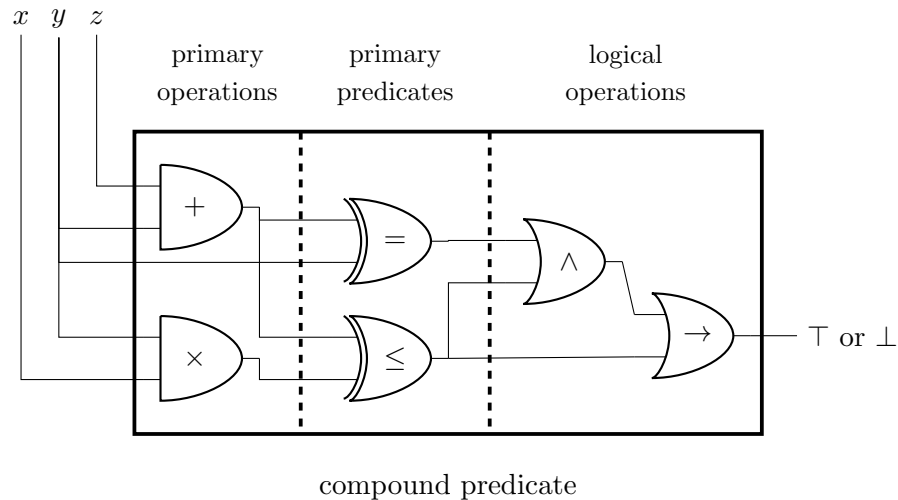


## DISCRETE MATH (MATH 2001)

### REVIEW SHEET II

#### V. Logic.

- (a) Structures.
- (b) Alphabet of symbols. Ingredients in a compound predicate.



- (c) Deciding the truth of a statement in a structure.
  - (i) Assigning tables to terms.
  - (ii) Assigning tables to atomic formulas.
  - (iii) Logical connectives. Truth Tables. Propositional tautology. Contradiction. Logical equivalence of propositions.
  - (iv) Disjunctive normal form.
  - (v) Prenex form.
  - (vi) Quantifier games. Winning strategies.
- (d) Proof.
  - (i) Definition of "proof".
  - (ii) Axioms. Tautology versus logically valid sentence.
  - (iii) Laws of deduction. Modus ponens, modus tollens.
  - (iv) Direct proof, proof of the contrapositive, and proof by contradiction. Proof by cases.
  - (v) The use of truth tables for designing proof strategies.
  - (vi) The relationship between truth and provability: semantic consequence ( $\Sigma \models S$ ) versus syntactic consequence ( $\Sigma \vdash S$ ).
  - (vii) Significance of Soundness, Completeness, and Decidability with regard to proof systems.
  - (viii) Relevance of the Church-Turing Theorem.
  - (ix) Relevance of Gödel's Completeness Theorem.

## VI. Counting.

- (a) Additive counting principle and multiplicative counting principle.
- (b) # functions  $f : k \rightarrow n$ . Characteristic functions,  $|\mathcal{P}(n)|$ .
- (c) # injective functions  $f : k \rightarrow n$ .
- (d) # bijective functions  $f : k \rightarrow n$ .
- (e) Binomial coefficients: definition, formula, recursion, Binomial Theorem, Pascal's Triangle.
- (f) Multinomial coefficients: definition, formula, recursion, Multinomial Theorem, Pascal's Pyramid.
- (g) Multichoose numbers: definition, formula.
- (h) Inclusion-exclusion.  $N_=(S) = \sum_{S \subseteq T \subseteq \mathcal{P}} (-1)^{|T|-|S|} N_{\geq}(T)$ .
- (i) # surjective functions  $f : k \rightarrow n$ .
- (j) Stirling numbers of the second kind: definition, formula, recursion, Binomial-type Theorem.

### General advice on preparing for a math test.

Be prepared to demonstrate understanding in the following ways.

- (i) Know the definitions of new concepts, and the meanings of the definitions.
- (ii) Know the statements and meanings of the major theorems.
- (iii) Know examples/counterexamples. (The purpose of an example is to illustrate the extent of a definition or theorem. The purpose of a counterexample is to indicate the limits of a definition or theorem.)
- (iv) Know how to perform the different kinds of calculations discussed in class.
- (v) Be prepared to prove elementary statements. (Understanding the proofs done in class is the best preparation for this.)
- (vi) Know how to correct mistakes made on old HW.

### Some definitions to know.

- (1) Predicate. Operation. Structure.
- (2) Logical connective. Truth table. Tautology. Contradiction.
- (3) Contrapositive. Converse. Inverse.
- (4) Disjunction. Conjunction. Disjunctive normal form. Conjunctive normal form.
- (5) Proof. Axioms. Rules of deduction.
- (6) Valid sentence.
- (7) Semantic consequence. Syntactic consequence.
- (8) Soundness. Completeness.
- (9) Additive counting principal. Multiplicative counting principal.
- (10) Binomial coefficient. Multinomial coefficient. Multichoose number. Stirling number.

### Some theorems to know.

- (1) Gödel's Completeness Theorem.
- (2) Binomial Theorem. Multinomial Theorem.
- (3) Inclusion-Exclusion Theorem.