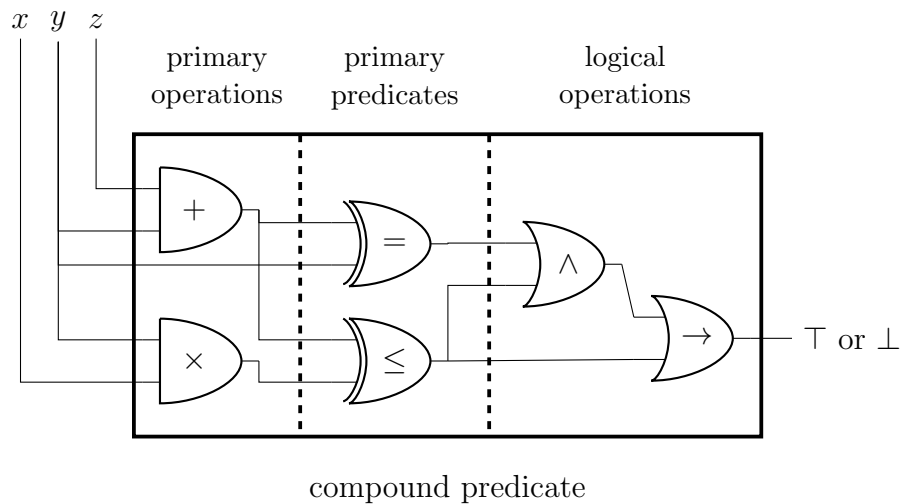


DISCRETE MATH (MATH 2001)

REVIEW SHEET II

V. Logic.

- (a) Structures.
- (b) Alphabet of symbols. Ingredients in a compound predicate.



- (c) Deciding the truth of a statement in a structure.
 - (i) Assigning tables to terms.
 - (ii) Assigning tables to atomic formulas.
 - (iii) Logical connectives. Truth Tables. Propositional tautology. Contradiction. Logical equivalence of propositions.
 - (iv) Disjunctive normal form.
 - (v) Prenex form.
 - (vi) Quantifier games. Winning strategies.
- (d) Proof.
 - (i) Definition of "proof".
 - (ii) Axioms. Tautology versus logically valid sentence.
 - (iii) Laws of deduction. Modus ponens, modus tollens.
 - (iv) Direct proof, proof of the contrapositive, and proof by contradiction. Proof by cases.
 - (v) The use of truth tables for designing proof strategies.
 - (vi) The relationship between truth and provability: semantic consequence ($\Sigma \models S$) versus syntactic consequence ($\Sigma \vdash S$).
 - (vii) Significance of Soundness, Completeness, and Decidability with regard to proof systems.

- (viii) Relevance of the Church-Turing Theorem.
- (ix) Relevance of Gödel's Completeness Theorem.

VI. Counting.

- (a) Additive counting principle and multiplicative counting principle.
- (b) # functions $f : k \rightarrow n$. Characteristic functions, $|\mathcal{P}(n)|$.
- (c) # injective functions $f : k \rightarrow n$.
- (d) # bijective functions $f : k \rightarrow n$.
- (e) Binomial coefficients: definition, formula, recursion, Binomial Theorem, Pascal's Triangle.
- (f) Multinomial coefficients: definition, formula, recursion, Multinomial Theorem, Pascal's Pyramid.
- (g) Multichoose numbers: definition, formula.
- (h) Inclusion-exclusion. $N_=(S) = \sum_{S \subseteq T \subseteq \mathcal{P}} (-1)^{|T|-|S|} N_>(T)$.
- (i) # surjective functions $f : k \rightarrow n$.
- (j) Stirling numbers of the second kind: definition, formula, recursion, Binomial-type Theorem.

General advice on preparing for a math test.

Be prepared to demonstrate understanding in the following ways.

- (i) Know the definitions of new concepts, and the meanings of the definitions.
- (ii) Know the statements and meanings of the major theorems.
- (iii) Know examples/counterexamples. (The purpose of an example is to illustrate the extent of a definition or theorem. The purpose of a counterexample is to indicate the limits of a definition or theorem.)
- (iv) Know how to perform the different kinds of calculations discussed in class.
- (v) Be prepared to prove elementary statements. (Understanding the proofs done in class is the best preparation for this.)
- (vi) Know how to correct mistakes made on old HW.