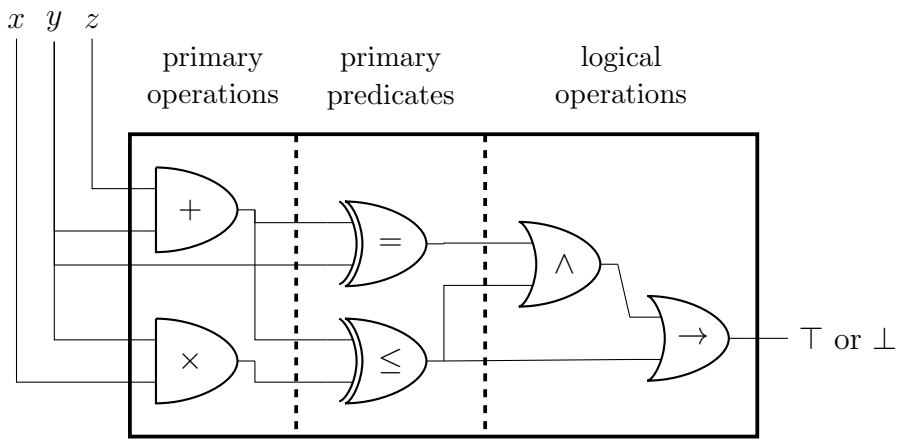
DISCRETE MATH (MATH 2001)

REVIEW SHEET II

V. Logic.

(a) Structures.

(b) Alphabet of symbols. Ingredients in a compound predicate.



compound predicate

(c) Deciding the truth of a statement in a structure.

- (i) Assigning tables to terms.
- (ii) Assigning tables to atomic formulas.
- (iii) Logical connectives. Truth Tables. Propositional tautology. Contradiction. Logical equivalence of propositions.
- (iv) Disjunctive normal form.
- (v) Prenex form.
- (vi) Quantifier games. Winning strategies.

(d) Proof.

- (i) Definition of "proof".
- (ii) Axioms. Tautology versus logically valid sentence.
- (iii) Laws of deduction. Modus ponens, modus tollens.
- (iv) Direct proof, proof of the contrapositive, and proof by contradiction. Proof by cases.
- (v) The use of truth tables for designing proof strategies.
- (vi) The relationship between truth and provability: semantic consequence ($\Sigma \models S$) versus syntactic consequence ($\Sigma \vdash S$).
- (vii) Significance of Soundness, Completeness, and Decidability with regard to proof systems.

(viii) Relevance of the Church-Turing Theorem.
(ix) Relevance of Gödel's Completeness Theorem.

VI. Counting.
(a) Additive counting principle and multiplicative counting principle.
(b) # functions $f : k \to n$. Characteristic functions, $|\mathcal{P}(n)|$.
(c) # injective functions $f : k \to n$.
(d) # bijective functions $f : k \to n$.
(e) Binomial coefficients: definition, formula, recursion, Binomial Theorem, Pascal's Triangle.
(f) Multinomial coefficients: definition, formula, recursion, Multinomial Theorem, Pascal's Pyramid.
(g) Multichoose numbers: definition, formula.
(h) Inclusion-exclusion. $N_=(S) = \sum_{S \subseteq T \subseteq \mathcal{P}} (-1)^{|T|-|S|} N_\geq(T)$.
(i) # surjective functions $f : k \to n$.
(j) Stirling numbers of the second kind: definition, formula, recursion, Binomial-type Theorem.

General advice on preparing for a math test.
Be prepared to demonstrate understanding in the following ways.

(i) Know the definitions of new concepts, and the meanings of the definitions.
(ii) Know the statements and meanings of the major theorems.
(iii) Know examples/counterexamples. (The purpose of an example is to illustrate the extent of a definition or theorem. The purpose of a counterexample is to indicate the limits of a definition or theorem.)
(iv) Know how to perform the different kinds of calculations discussed in class.
(v) Be prepared to prove elementary statements. (Understanding the proofs done in class is the best preparation for this.)
(vi) Know how to correct mistakes made on old HW.