Solutions to HW 6.

1. Write the following propositions in disjunctive normal form, assuming that each proposition is a function of p, q and r.

   (i) $p \rightarrow r$

$((\neg p) \wedge (\neg q) \wedge (\neg r)) \vee ((\neg p) \wedge (\neg q) \wedge r) \vee ((\neg p) \wedge q \wedge (\neg r)) \vee ((\neg p) \wedge q \wedge r) \vee (p \wedge (\neg q) \wedge r) \vee (p \wedge q \wedge r)$

   (ii) $((p \rightarrow q) \rightarrow ((\neg p) \leftrightarrow r))$.

$((\neg p) \wedge (\neg q) \wedge r) \vee ((\neg p) \wedge q \wedge r) \vee (p \wedge (\neg q) \wedge (\neg r)) \vee (p \wedge (\neg q) \wedge r) \vee (p \wedge q \wedge (\neg r))$

   (iii) q

$((\neg p) \wedge q \wedge (\neg r)) \vee ((\neg p) \wedge q \wedge r) \vee (p \wedge q \wedge (\neg r)) \vee (p \wedge q \wedge r)$

2. Write the following axioms of set theory as formal sentences.

   (i) Extensionality.

$$(\forall A)(\forall B)((A = B) \leftrightarrow (\forall z)((z \in A) \leftrightarrow (z \in B)))$$

   (ii) Pairing.

$$(\forall A)(\forall B)(\exists P)(\forall z)((z \in P) \leftrightarrow ((z = A) \vee (z = B)))$$

   (iii) Power set.

$$(\forall A)(\exists P)(\forall z)((z \in P) \leftrightarrow \underline{(z \subseteq A)})$$

   or

$$(\forall A)(\exists P)(\forall z)((z \in P) \leftrightarrow \underline{((\forall w)((w \in z) \rightarrow (w \in A)))})$$

3. In 1959, Pete Seeger took lines from the Book of Ecclesiastes to write a song, which was made famous by the Byrds in 1965. One line is:

   To every thing there is a season, and a time for every purpose under heaven.

   Write this as a formal sentence using predicates $S(s,t) = $ "s is the season for thing t" and $P(T,p) = $ "T is the time for purpose p".

$$((\forall t)(\exists s)S(s,t)) \wedge ((\forall p)(\exists T)P(T,p))$$