

The Axioms of Set Theory.

Equality

- (1) (Extensionality) Two sets are equal if they have the same elements.

Existence of Special Sets

- (2) (Empty Set) There is a set with no elements.
- (3) (Infinity) There is an inductive set.

Creation of New Sets

- (4) (Pairing) Given sets A and B , there is a set P whose elements are A and B . (I.e., the unordered pair $P = \{A, B\}$ is a set.)
- (5) (Union) Given a set A , there is a set $\bigcup A$ whose elements are the elements of elements of A .
- (6) (Power Set) Given a set A , there is a set $\mathcal{P}(A)$ whose elements are the subsets of A .
- (7) (Separation) Given a set A and a property given by a formula φ , there is a set whose elements are the elements of A that satisfy φ . ($\{x \in A \mid \varphi(x)\}$ is a set.)
- (8) (Replacement) Given a set A and a function F given by a formula, then $\{F(x) \mid x \in A\}$ is a set.
- (9) (Choice) Given $A = \{X_i \mid i \in I\}$, a set of nonempty pairwise-disjoint sets, there is a set C that intersects each X_i in exactly one element.

Sets have Special Properties

- (10) (Foundation) If A is a nonempty set, then there is an $x \in A$ such that x and A are disjoint. x is called an \in -minimal element of A .