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$Th(\mathbb{Q})$ has Continuumly many countable models up to isomorphism.

Proof. Define $q_1 < q_2$ by the formula $\varphi_<(q_1, q_2)$ which is
$\exists x_1 \exists x_2 \exists x_3 \exists x_4 \left(x_1 \neq 0 \ \wedge \ q_2 - q_1 = x_1^2 + x_2^2 + x_3^2 + x_4^2\right)$. This is the standard order on $\mathbb{Q}$ because every positive element in $\mathbb{Q}$ can be written as the sum of 4 squares.

For each $r \in \mathbb{R}$ define the partial type $r < a_n^r \wedge b_n^r < r$ where a_n^r is a monotonically decreasing sequence in $\mathbb{Q}$ converging to r and b_n^r is a monotonically increasing sequence in $\mathbb{Q}$ converging to r. Each of these partial 1-types can be extended to a complete type by stating how r relates to every other rational number. Therefore there are continuumly many 1-types in the $Th(\mathbb{Q})$.

A theory with continuumly many 1-types has continuumly many countable models up to isomorphism.

For each type there is a countable model that realizes that type, and a countable model can only realize countably many types. Therefore there are continuumly many models, each of these models will not be isomorphic because there are types realized in one that are not realized in the other. $\square$