Connor Meredith
Vishnu Murali
Mateo Muro

11. Show that if $\mathbb{A}$ and $\mathbb{B}$ are nonisomorphic finite L-structures, then there is a sentence true in $\mathbb{A}$ that is false in $\mathbb{B}$.

Proof. We first consider the case where the cardinalities of A and B are m and n such that $m \neq n$.
Then one of the following sentences must hold (The first when the size of A is less than that of B and the other when the size of A is greater)
$\phi = \neg((\exists x_1), (\exists x_2), ..., (\exists x_m), (\exists x_{m+1}) \bigwedge_{1 \leq j < i \leq m+1} x_i \neq x_j)$, which basically asserts that there are no $m+1$ distinct elements in A.
Similarly for the other case $(m > n)$,
$\phi = (\exists x_1), (\exists x_2), .., (\exists x_n), (\exists x_{n+1}), \bigwedge_{1 \leq j < i \leq n+1} (x_i \neq x_j)$.

Now we consider the case when they have the same cardinality (n),and define the set of all bijections from $\mathbb{A} \to \mathbb{B}$ as H.
Then, for every bijection $h \in H$, there exists some constant c_h, or a function F_h, or a relation R_h not preserved by h.

Let L' be the language in the signature that involves these symbols only. L' is a sublanguage of L, it is defined in a finite signature, and it was selected so that no $h \in H$ is an L'-isomorphism from the reduct $\mathbb{A}|_{L'}$ to the reduct $\mathbb{B}|_{L'}$.

Since L' is defined in a finite signature, the solution to modth1p10 establishes that there is an L'-sentence $\phi_{\mathbb{A}}$ which defines $\mathbb{A}|_{L'}$ up to isomorphism. This sentence will be true in $\mathbb{A}$ but false in $\mathbb{B}$.

Thus we can always find a sentence ϕ such that $\mathbb{A} \models \phi$, but $\mathbb{B} \not\models \phi$.