MODEL THEORY
HOMEWORK ASSIGNMENT I

Read Chapter 1.

PROBLEMS

1. (Toby Aldape, Raymond Baker, Chris Eblen) Show that the following pairs of abelian groups are not elementarily equivalent.

(a) $\mathbb{Z}$ and $\mathbb{Q}$
(b) $\mathbb{Z}$ and $\mathbb{Z} \times \mathbb{Z}$

2. (Toby Aldape, Raymond Baker, Chris Eblen) Show that the class of simple groups is not elementary.

3. (Toby Aldape, Raymond Baker, Chris Eblen) Show that the theory of finite groups is different than the theory of all groups.

4. (Hayden Hollis, Howie Jordan, Sangman Lee) Exercise 1.4.3 of the text.

5. (Hayden Hollis, Howie Jordan, Sangman Lee) Let σ be a signature with one binary relation symbol and no other symbols. Show that there are exactly 2^κ nonisomorphic σ-structures of cardinality κ for each infinite κ.

6. (Hayden Hollis, Howie Jordan, Sangman Lee) Let L be a language in a finite signature. Show that there are at least $\aleph_0$-many distinct (closed) L-theories, and at most $2^{\aleph_0}$-many (closed) distinct L-theories.

7. (Thomas Magnuson, Trevor Manders, Chase Meadors) Show that the following classes of graphs are elementary.

(a) The class of infinite graphs.
(b) The class of triangle-free graphs.
(c) The class of 2-colorable graphs.

8. (Thomas Magnuson, Trevor Manders, Chase Meadors) Show that the class of connected graphs is not elementary.

9. (Thomas Magnuson, Trevor Manders, Chase Meadors) Show that the class of well ordered sets is not elementary.

10. **(Connor Meredith, Vishnu Murali, Mateo Muro) Exercise 1.4.2(b) of the text.**

11. **(Connor Meredith, Vishnu Murali, Mateo Muro) Show that if $\mathbb{A}$ and $\mathbb{B}$ are nonisomorphic finite L-structures, then there is a sentence true in $\mathbb{A}$ that is false in $\mathbb{B}$.**

12. **(Connor Meredith, Vishnu Murali, Mateo Muro) Explain why the following sets are definable in $\mathbb{R} = \langle\{\text{reals}\}; \cdot, +\rangle$.**

- **(a) The unit interval.**
- **(b) $\{n\}$ for any integer n.**
- **(c) $\{x\}$ for any real algebraic number x.**

13. **(Andrew Stocker, Patrick Wynne) Give an example of a nondefinable subset of each of the following structures.**

- **(a) An infinite "pure set". (This is a structure in the "empty signature", meaning the signature with no nonlogical symbols.)**
- **(b) The abelian group $\mathbb{Z} = \langle \mathsf{integers}; +, -, 0\rangle$.**
- **(c) The field $\mathbb{C} = \langle \mathsf{complex\ numbers}; +, -, 0, \cdot, 1\rangle$.**

14. **(Andrew Stocker, Patrick Wynne) Show that $(\exists x_1)(\forall x_2)P(x_1, x_2) \rightarrow (\forall x_2)(\exists x_1)P(x_1, x_2)$ is logically valid, but not a tautology.**