

Discrete Math  
Quiz 4

Name: \_\_\_\_\_

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Define a function  $f : \mathbb{N} \rightarrow \mathbb{N}$  by the following recursion

$$\begin{aligned} f(0) &= 1 \\ f(n+1) &= 2 \cdot f(n). \end{aligned}$$

Use induction to prove that  $f(k) = 2^k$  for all  $k \in \mathbb{N}$ . (You may use any laws of arithmetic you know.)

We prove this by induction on  $k$ .

(Base Case:  $k = 0$ )  $f(0) = 1 = 2^0$ . ✓

(Inductive Step: Assume  $f(k) = 2^k$ , prove  $f(k+1) = 2^{k+1}$ )

$$\begin{aligned} f(k+1) &= 2 \cdot f(k) && (\text{RR}, f) \\ &= 2 \cdot 2^k && (\text{IH}) \\ &= 2^{k+1}. \quad \checkmark && (\text{arithmetic}) \end{aligned}$$