

Discrete Math
Quiz 4

Name: _____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Define a function $f : \mathbb{N} \rightarrow \mathbb{N}$ by the following recursion

$$\begin{aligned} f(0) &= 1 \\ f(n+1) &= 2 \cdot f(n). \end{aligned}$$

Use induction to prove that $f(k) = 2^k$ for all $k \in \mathbb{N}$. (You may use any laws of arithmetic you know.)

We prove this by induction on k .

(Base Case: $k = 0$) $f(0) = 1 = 2^0$. ✓

(Inductive Step: Assume $f(k) = 2^k$, prove $f(k+1) = 2^{k+1}$)

$$\begin{aligned} f(k+1) &= 2 \cdot f(k) && \text{(RR, } f) \\ &= 2 \cdot 2^k && \text{(IH)} \\ &= 2^{k+1}. \quad \checkmark && \text{(arithmetic)} \end{aligned}$$