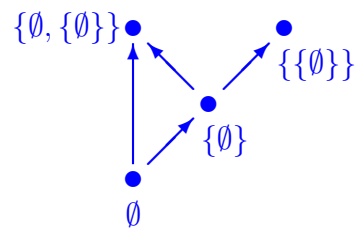
Solutions to HW 1.

1. Define $V_0 = \emptyset$, $V_1 = \mathcal{P}(V_0)$, $V_2 = \mathcal{P}(V_1)$, $V_3 = \mathcal{P}(V_2)$, and so on.
   (a) List the elements of V_0, V_1, V_2 and V_3.
      (i) $V_0 = \emptyset$,
      (ii) $V_1 = \mathcal{P}(\emptyset) = \{\emptyset\}$,
      (iii) $V_2 = \mathcal{P}(V_1) = \{\emptyset, \{\emptyset\}\}$,
      (iv) $V_3 = \mathcal{P}(V_2) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$,
   (b) Draw a directed graph whose "dots" are the sets in V_3 and where $x \to y$ means $x \in y$. (Hint: your graph should have four "dots" and four edges.)



2. Find sets A and B satisfying the given conditions.
   (a) $A \in B$ and $A \not\subseteq B$.
   There are many answers, such as $A = \{0\}$ and $B = \{\{0\}\}$.
   (b) $A \in B$ and $A \subseteq B$.
   You could take $A = \{0\}$ and $B = \{0, \{0\}\}$.
   (c) $A \notin B$ and $A \subseteq B$.
   You could take $A = B = \emptyset$.

3. Show that $\bigcup \mathcal{P}(x) = x$.

To shows that the two sets, $\bigcup \mathcal{P}(x)$ and x, are equal, we must show that they have the same elements.

Part 1: We show that any element $z \in x$ is an element of $\bigcup \mathcal{P}(x)$.

Choose any $z \in x$. Then $\{z\}$ is a set, by the Pairing Axiom, and $\{z\} \subseteq x$ by the definition of $\subseteq$, so $\{z\} \in \mathcal{P}(x)$, according to the definition of $\mathcal{P}(x)$. But now $z \in \{z\} \in \mathcal{P}(x)$, so z is "2 levels down" from $\mathcal{P}(x)$, which implies that $z \in \bigcup \mathcal{P}(x)$.

Part 2: We show that any element $z \in \bigcup \mathcal{P}(x)$ is an element of x.

Now choose any $z \in \bigcup \mathcal{P}(x)$. This means that z is "2 levels down" from $\mathcal{P}(x)$, so there is some y such that $z \in y \in \mathcal{P}(x)$. Since $y \in \mathcal{P}(x)$ we know that $y \subseteq x$, by the definition of $\mathcal{P}(x)$. But now we know that $z \in y$ and $y \subseteq x$, so we derive $z \in x$ from the definition of $\subseteq$.