

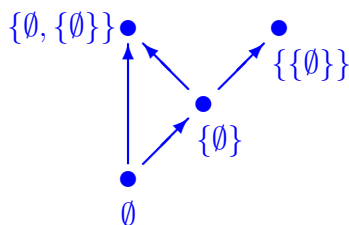
## Solutions to HW 1.

1. Define  $V_0 = \emptyset$ ,  $V_1 = \mathcal{P}(V_0)$ ,  $V_2 = \mathcal{P}(V_1)$ ,  $V_3 = \mathcal{P}(V_2)$ , and so on.

(a) List the elements of  $V_0, V_1, V_2$  and  $V_3$ .

- (i)  $V_0 = \emptyset$ ,
- (ii)  $V_1 = \mathcal{P}(\emptyset) = \{\emptyset\}$ ,
- (iii)  $V_2 = \mathcal{P}(V_1) = \{\emptyset, \{\emptyset\}\}$ ,
- (iv)  $V_3 = \mathcal{P}(V_2) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$ ,

(b) Draw a directed graph whose “dots” are the sets in  $V_3$  and where  $x \rightarrow y$  means  $x \in y$ . (Hint: your graph should have four “dots” and four edges.)



2. Find sets  $A$  and  $B$  satisfying the given conditions.

(a)  $A \in B$  and  $A \not\subseteq B$ .

There are many answers, such as  $A = \{0\}$  and  $B = \{\{0\}\}$ .

(b)  $A \in B$  and  $A \subseteq B$ .

You could take  $A = \{0\}$  and  $B = \{0, \{0\}\}$ .

(c)  $A \notin B$  and  $A \subseteq B$ .

You could take  $A = B = \emptyset$ .

3. Show that  $\bigcup \mathcal{P}(x) = x$ .

To show that the two sets,  $\bigcup \mathcal{P}(x)$  and  $x$ , are equal, we must show that they have the same elements.

**Part 1:** We show that any element  $z \in x$  is an element of  $\bigcup \mathcal{P}(x)$ .

Choose any  $z \in x$ . Then  $\{z\}$  is a set, by the Pairing Axiom, and  $\{z\} \subseteq x$  by the definition of  $\subseteq$ , so  $\{z\} \in \mathcal{P}(x)$ , according to the definition of  $\mathcal{P}(x)$ . But now  $z \in \{z\} \in \mathcal{P}(x)$ , so  $z$  is “2 levels down” from  $\mathcal{P}(x)$ , which implies that  $z \in \bigcup \mathcal{P}(x)$ .

**Part 2:** We show that any element  $z \in \bigcup \mathcal{P}(x)$  is an element of  $x$ .

Now choose any  $z \in \bigcup \mathcal{P}(x)$ . This means that  $z$  is “2 levels down” from  $\mathcal{P}(x)$ , so there is some  $y$  such that  $z \in y \in \mathcal{P}(x)$ . Since  $y \in \mathcal{P}(x)$  we know that  $y \subseteq x$ , by the definition of  $\mathcal{P}(x)$ . But now we know that  $z \in y$  and  $y \subseteq x$ , so we derive  $z \in x$  from the definition of  $\subseteq$ .