

The Axioms of Set Theory.

Equality

- (1) (Extensionality) Two sets are equal if they have the same elements.

Existence of Special Sets

- (2) (Empty Set) There is a set with no elements.
- (3) (Infinity) There is an inductive set.

Creation of New Sets

- (4) (Pairing) If A and B are sets, then $\{A, B\}$ is a set.
- (5) (Union) If A is a set, then the collection of all elements of elements of A is a set. It is denoted $\bigcup A$ and called the union of A .
- (6) (Power Set) If A is a set, then the collection of subsets of A is a set. It is denoted $\mathcal{P}(A)$ and called the power set of A .
- (7) (Separation) If A is a set and P is a property given by a formula, then $\{x \in A \mid P(x)\}$ is a set.
- (8) (Replacement) If A is a set and F is a function given by a formula, then $\{F(x) \mid x \in A\}$ is a set.
- (9) (Choice) If $A = \{X_i \mid i \in I\}$ is set of nonempty pairwise disjoint sets, then there is a set C that intersects each X_i in exactly one element.

Sets have Special Properties

- (10) (Foundation) If A is a nonempty set, then there is an $x \in A$ such that x and A are disjoint. x is called an \in -minimal element of A .