

# MODEL THEORY

## HOMEWORK ASSIGNMENT III

Read Chapter 3.

### PROBLEMS

1. (Lotfi, Sparks) You might conjecture that the composition of elementary embeddings is elementary, or that the composition of nonelementary embeddings is nonelementary. This problem explores all relationships of this type. Namely it explores which maps, in a triple of embeddings  $(f, g, g \circ f)$ , can be (or are forced to be) elementary.

For each triple  $(x, y, z) \in \{\text{elementary, not}\}^3$  find an example

$$\mathbb{A} \xrightarrow{f} \mathbb{B} \xrightarrow{g} \mathbb{C}$$

realizing the triple, if possible, or explain why there is no example.

2. (Jack, Tanksalvala) Suppose that  $\mathcal{L}$  is a language and  $\mathcal{L}'$  is an expansion of  $\mathcal{L}$  by some set  $C$  of additional constant symbols. Suppose that  $T$  is an  $\mathcal{L}$ -theory that has quantifier elimination and that  $T' \supseteq T$  is an  $\mathcal{L}'$ -theory extending  $T$ . Show that  $T'$  has q.e.

The theory of dense linear order without endpoints has q.e. (you may assume this). Show that any theory of dense linear order with some additional constant symbols is complete iff the theory completely decides how the order relation restricts to the interpretations of the constant symbols.

3. (Lotfi, Sparks) Let  $t(n)$  be the least number  $N$  such that there is some infinite structure with exactly  $N$   $n$ -types over the empty set. (So  $t(1) = 1, t(2) = 2, t(3) = 5, \dots$ ) Show that there exists a single infinite structure with exactly  $t(n)$   $n$ -types for each  $n$ . Classify all such structures.

4. (Jack, Tanksalvala) Let  $T$  be the theory of  $\mathbb{A} = \langle \omega; \cdot, + \rangle$ , and show that  $|S_1(T)| = 2^\omega$ . Conclude that there are  $2^\omega$  countable models of  $T$  up to isomorphism.