# MODEL THEORY
# HOMEWORK ASSIGNMENT I

Read Chapter 1.

## PROBLEMS

**1. (Berg, Jack) Let $\sigma$ be a signature with one binary relation symbol and no other symbols. Show that there are exactly $2^\kappa$ nonisomorphic $\sigma$-structures of cardinality $\kappa$ for each infinite $\kappa$.**

**2. (Lessard, Lotfi) Show that if $\alpha\colon \mathbf{A} \to \mathbf{B}$ is an isomorphism, $\varphi$ is a formula, and $v$ is a valuation, then $\mathbf{A} \models \varphi[v]$ iff $\mathbf{B} \models \varphi[\alpha(v)]$.**

**3. (Shriner, Sparks, Tanksalvala) Show that the following pairs of abelian groups are not elementarily equivalent.**

(a) $\mathbb{Z}$ **and** $\mathbb{Q}$
(b) $\mathbb{Z}$ **and** $\mathbb{Z} \times \mathbb{Z}$

**4. (Berg, Jack) Show that the class of simple groups is not elementary. (Use the language of groups.)**

**5. (Lessard, Lotfi) Let $\kappa < \lambda < \mu$ be infinite cardinals. Give an example of a structure of cardinality $\mu$ that has a substructure of cardinality $\kappa$ but no substructure of cardinality $\lambda$.**

**6. (Shriner, Sparks, Tanksalvala) Show that the class of well ordered sets is not elementary. (Use the language of partially ordered sets.)**

**7. (Shriner, Sparks, Tanksalvala) If $\mathbf{A}$ is a structure, then a subset $U \subseteq A$ is definable if there is some formula $\varphi(x)$ such that $u \in U$ iff $\mathbf{A} \models \varphi[u]$. Explain why the following sets are definable in $\mathbb{R} = \langle\{\text{reals}\}; \cdot, +\rangle$.**

(a) **The unit interval.**
(b) $\{n\}$ **for any integer** $n$.
(c) $\{x\}$ **for any real algebraic number** $x$.