Given any proper chain of infinite cardinals $\kappa < \lambda < \mu$ we may construct a structure of cardinality μ which admits a substructure of cardinality κ but admits no substructure of cardinality λ.

Let $\kappa < \lambda < \mu$ be given, let C be a set of cardinality μ. Let $B \subset C$ be a subset of cardinality λ, and let $A \subset B$ be a subset of cardinality κ. Then the structure

$$\mathbf{C} = \left\langle C; \left\{ f_c\left(x\right) = \begin{cases} x & x \in A \\ c & c \notin A \end{cases} \middle| \, c \in C \right\} \right\rangle$$

exhibits the desired property. We note that $\mathbf{A} = \left\langle A; \left\{f_c\right\}_{c \in C} \right\rangle$ is a substructure of cardinality κ but if $\mathbf{B}'$ is to be any substructure of cardinality greater than κ, its domain must contain at least one element not in A, and closing that domain under the functions results in C, which is of cardinality μ. No intermediate substructures may then exist.