1. Let σ be a signature with one binary relation symbol, R, and no other symbols. We will show that there are exactly 2^κ nonisomorphic σ-structures of cardinality κ for each infinite κ. Since σ contains only a single binary relation, R, and no functions, a structure, (M, R), with signature σ is uniquely determined by the pairs of elements contained in R. Since κ is infinite, $|M^2| = \kappa^2 = \kappa$, and any element of M^2 is either contained in R or $M^2 \setminus R$. Therefore, we can think of each structure as a function from κ to 2, corresponding to whether each pair is in the structure's interpretation of R, and thus 2^κ is the maximal number of structures we could define.

   It remains to be shown that there are exactly 2^κ distinct structures up to isomorphism. We do this by encoding the subsets of κ and the well-ordering of κ into a binary relation. Let $<$ denote the standard well-ordering of the elements of κ. We consider structures of the form $\mathcal{M} = (\kappa, R_A)$, where R_A is a shorthand convention indicating that for some $A \subseteq \kappa$,

$$R_A^{\mathcal{M}} = \{(\alpha, \alpha) \ : \ \alpha \in A\} \cup \{(\gamma, \delta) \ : \ \gamma < \delta\}.$$

   Clearly, all such $\mathcal{M}$ are of size κ – it therefore remains to be shown that there are 2^κ such structures which are not isomorphic.

   It will be sufficient to demonstrate that if $A \neq B$, then $\mathcal{M}_A = (\kappa, R_A) \ncong \mathcal{M}_B = (\kappa, R_B)$. Without loss of generality, suppose $\gamma \in A$ with $\gamma \notin B$. In anticipation of a contradiction, assume $\phi : \mathcal{M}_A \to \mathcal{M}_B$ is an isomorphism. We claim that ϕ can be thought of as an order isomorphism from κ to κ – by construction ϕ is bijective, so it will suffice to show that, given $\alpha, \beta \in \kappa$, $\alpha < \beta$ if and only if $\phi(\alpha) < \phi(\beta)$. Suppose $\alpha < \beta$ – then $(\alpha, \beta) \in R_A^{\mathcal{M}_A}$ and thus $(\phi(\alpha), \phi(\beta)) \in R_B^{\mathcal{M}_B}$. Since $\alpha \neq \beta$ and ϕ is a bijection, we must have that $\phi(\alpha) \neq \phi(\beta)$. Consequently, $(\phi(\alpha), \phi(\beta)) \in R_B^{\mathcal{M}_B}$ implies that $\phi(\alpha) < \phi(\beta)$. Suppose instead that $\phi(\alpha) < \phi(\beta)$ – then $(\phi(\alpha), \phi(\beta)) \in R_B^{\mathcal{M}_B}$ and thus $(\alpha, \beta) \in R_A^{\mathcal{M}_A}$. Since $\phi(\alpha) \neq \phi(\beta)$ and ϕ is a bijection, we must have that $\alpha \neq \beta$. Consequently, $(\alpha, \beta) \in R_A^{\mathcal{M}_A}$ implies that $\alpha < \beta$. It therefore is indeed the case that ϕ can be thought of as an order isomorphism. Consequently, γ is the unique element of height γ in A, and so $\phi(\gamma)$ must be the unique element of height γ in B. Therefore, $\phi(\gamma) = \gamma$, and so $\gamma \in B$. This is a contradiction, and thus if $A \neq B$, then $\mathcal{M}_A = (\kappa, R_A) \ncong \mathcal{M}_B = (\kappa, R_B)$. Since there are 2^κ distinct subsets of κ, we have produced 2^κ structures which are not isomorphic.