

Linear Algebra
Quiz 7

Name: _____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. $\mathcal{B} = (1, t, t^2)$ and $\mathcal{C} = (1, 1 + t, 1 + t + t^2)$ are bases for $\mathbb{P}_2(t)$.

(a) Find the change of basis matrix from \mathcal{B} to \mathcal{C} (i.e. ${}_C[I]_B$).

$${}_C[I]_B = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) Find the change of basis matrix from \mathcal{C} to \mathcal{B} .

$${}_B[I]_C = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

(c) The matrix for differentiation written in basis \mathcal{B} is ${}_B[D]_B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$. Use your answer to (b) to find the matrix for differentiation written in basis \mathcal{C} .

$$\begin{aligned} {}_C[D]_C &= {}_C[I]_B \cdot {}_B[D]_B \cdot {}_B[I]_C \\ &= \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$