

Linear Algebra

Quiz 3

Name: _____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. True or false? an $n \times n$ nonsingular matrix is row equivalent to the $n \times n$ identity matrix. (Explain your answer.)

Solution 1: True! If A is nonsingular (=invertible), then its inverse can be computed by GJE: $[A|I] \rightarrow [I|A^{-1}]$. If you restrict attention to what is happening left of the bar, then you see that row operations are converting A to I . Therefore A is row equivalent to I .

Solution 2: True! Any invertible $n \times n$ matrix A is a product of elementary matrices, say $A = E_1 \cdots E_n$. Since $A = E_1 \cdots E_n I$ the matrix A may be obtained from I through row operations.

2. Recall that a linear transformation between vector spaces is defined to be a function $T: V \rightarrow W$ that *preserves the additive and scaling structure*. What exactly does the italicized phrase mean?

- $T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$,
- $T(-\mathbf{u}) = -T(\mathbf{u})$,
- $T(\mathbf{0}) = \mathbf{0}$, and
- $T(r\mathbf{u}) = rT(\mathbf{u})$.