Practice

(1) (Find the determinant)

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (2 different ways).

(b) $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 2 & 4 \end{bmatrix}$

(2) (Find the determinant by inspection)

(a) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 4 & 5 & 6 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$

(3) (Find the characteristic polynomial)

(a) $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix}$

(c) $\left[\begin{array}{cc|c} a & b & 0 \\ c & d & 0 \\ \hline 0 & 0 & r \end{array}\right]$

(4) (Find the e-values and e-spaces)

(a) $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

(b) $\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \hline 0 & 0 & 2 \end{array}\right]$

(c) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

Some basic facts about determinants.

(1) $\det(A^T) = \det(A)$.
(2) $\det(AB) = \det(A) \cdot \det(B)$, $\det(I) = 1$, and $\det(A^{-1}) = \frac{1}{\det(A)}$.
(3) $A \cdot \mathrm{adj}(A) = \det(A) \cdot I$. A matrix A is invertible iff $\det(A) \neq 0$, and when this condition holds we have $A^{-1} = \frac{1}{\det(A)}\mathrm{adj}(A)$.
(4) If $V = \mathbb{R}^n$, then $\det\colon V^n \to \mathbb{R}$ is the unique alternating n-linear form whose value at I is 1.
(5) The steps of Gaussian elimination affect the determinant in the following way:
  (a) scaling a row by r scales the determinant by r,
  (b) interchanging two rows changes the sign of the determinant, and
  (c) adding a multiple of a row to a different row has no effect on the determinant.
(6) If $A = \left[\begin{array}{c|c} A_{1,1} & A_{1,2} \\ \hline 0 & A_{2,2} \end{array}\right]$ is block upper triangular, then $\det(A)$ equals the product $\det(A_{1,1}) \cdot \det(A_{2,2})$ of the determinants of the diagonal blocks.