

## Practice about matrices for transformations

- (1) (a) Draw each of the following bases for  $\mathbb{R}^2$  on its own copy of the plane:  $\mathcal{E} = (\mathbf{e}_1, \mathbf{e}_2) = \left( \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$ ,  $\mathcal{B} = (\mathbf{v}_1, \mathbf{v}_2) = \left( \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right)$ , and  $\mathcal{C} = (\mathbf{w}_1, \mathbf{w}_2) = \left( \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right)$ .

- (b) Find the  $\mathcal{E}$ -coordinates,  $\mathcal{B}$ -coordinates, and  $\mathcal{C}$ -coordinates for the vector  $\mathbf{u} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ .

- (c) Find the change of basis matrix  ${}_C[I]_B$  and verify that  ${}_C[I]_B[\mathbf{u}]_B = [\mathbf{u}]_C$ .

- (2)  $\mathcal{B} = (1, t, t^2, t^3)$  and  $\mathcal{C} = (1, t - 1, (t - 1)^2, (t - 1)^3)$  are ordered bases for  $\mathbb{P}_3(t)$ . Find the change of basis matrices  ${}_C[I]_B$  and  ${}_B[I]_C$ .

(3) In this problem use the basis  $\mathcal{B} = (1, t, t^2, \dots, t^m)$  for the polynomial space  $\mathbb{P}_m(t)$ .

(a) Find the matrix for differentiation  $D: \mathbb{P}_n(t) \rightarrow \mathbb{P}_{n-1}(t): f(t) \mapsto f'(t)$ .

(b) Find the matrix for integration  $J: \mathbb{P}_n(t) \rightarrow \mathbb{P}_{n-1}(t): f(t) \mapsto \int_0^t f(x) dx$ .

(c) Use the results from parts (b) and (c) to show that  $D \circ J = I$  (the identity transformation), but  $J \circ D \neq I$ .

(d) Find a basis for the kernel of  $JD$ .