Practice Problems About N(A) and R(A)

Recall: $N(A)$ = the nullspace of A, $R(A)$ = the column space of A, and $\text{rank}(A) = \dim(R(A))$.

(1) Find the column space and nullspace of the following matrix:

$$A = \begin{bmatrix} 1 & 3 & 2 & -6 \\ 3 & 9 & 1 & 5 \\ 2 & 6 & -1 & 9 \\ 5 & 15 & 0 & 14 \end{bmatrix}$$

(2) Find a basis for the subspace spanned by the following vectors.

$$\begin{bmatrix} 1 \\ -1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ -7 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ -7 \\ 6 \\ -9 \end{bmatrix}.$$

(3) What is the rank of a 6×8 matrix whose nullspace has dimension 3?

(4) Construct a 4×3 matrix of rank 1.

(5) True or False? Justify your answer.

(a) If a set of p vectors span a p-dimensional subspace U, then $\dim(U) = p$.

(b) If A is $m \times n$, then the dimensions of the column space and the nullspace add up to n.

(c) The columns of an invertible $n \times n$ matrix form a basis for $\mathbb{R}^n$.

(d) The column space of A is the set of solutions to $A\mathbf{x} = \mathbf{b}$.

(e) Row operations do not affect the dependence relations among the columns of A.