Linear Algebra
Quiz 9

Name:____________________________

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. (Every invertible matrix is a change-of-basis matrix.) Let A be an arbitrary invertible $n \times n$ real matrix. Explain why there exist bases $\mathcal{B}$ and $\mathcal{C}$ for $\mathbb{R}^n$ such that $A = {}_{\mathcal{C}}[I]_{\mathcal{B}}$ is the change-of-basis matrix from $\mathcal{B}$ to $\mathcal{C}$.

We must show that if A is invertible, then there exist bases $\mathcal{B} = (\mathbf{b}_1, \ldots, \mathbf{b}_n)$ and $\mathcal{C} = (\mathbf{c}_1, \ldots, \mathbf{c}_n)$ such that $A = {}_{\mathcal{C}}[I]_{\mathcal{B}}$.

Solution 1: We want $A = [[\mathbf{b}_1]_{\mathcal{C}}, \ldots, [\mathbf{b}_n]_{\mathcal{C}}]$. If we take $\mathcal{C} = (\mathbf{e}_1, \ldots, \mathbf{e}_n)$ to be the standard basis for $\mathbb{R}^n$, then $\mathcal{B}$ must be chosen so that so that $A = [\mathbf{b}_1, \ldots, \mathbf{b}_n]$. This can be accomplished by taking $\mathcal{B}$ to be the sequence of columns of A.

Solution 2: We want to choose $\mathcal{B}$ and $\mathcal{C}$ so that, after applying Gaussian elimination to $[\,\mathcal{C} \,|\, \mathcal{B}\,]$ we obtain $[\,I \,|\, A\,]$. This can be accomplished by choosing $\mathcal{C}$ to be sequence of columns of I (that is, $\mathcal{C}$ is the standard basis) and $\mathcal{B}$ to be the sequence of columns of A.