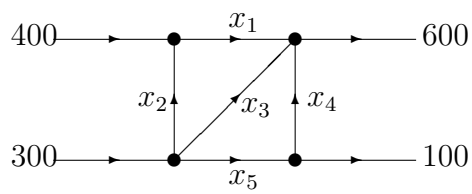
Linear Algebra
Quiz 5

Name:_____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. The flow of traffic in vehicles per hour through a network of streets is shown in the figure.



(a) Solve the associated system of linear equations.

(b) What is the largest physically possible value for x_3?

To create the system note that the flow into each node must equal the flow out, hence

$$\begin{array}{rrrrrl} x_1 & -x_2 & & & & = 400 \\ x_1 & & +x_3 & +x_4 & & = 600 \\ & x_2 & +x_3 & & +x_5 & = 300 \\ & & & -x_4 & +x_5 & = 100 \end{array}$$

Solving yields

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 700 - x_3 - x_5 \\ 300 - x_3 - x_5 \\ x_3 \\ x_5 - 100 \\ x_5 \end{bmatrix} = \begin{bmatrix} 700 \\ 300 \\ 0 \\ -100 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -1 \\ -1 \\ 0 \\ 1 \\ 1 \end{bmatrix},$$

which is the answer to (a).

For the answer to (b), note that for flow x_4 to be in the indicated direction we must have $x_5 \geq 100$. Then for flow x_2 to be in the indicated direction we must have $0 \leq x_2 = 300 - x_3 - x_3 \leq 200 - x_3$, so $x_3 \leq 200$. But $x_3 = 200$ can be achieved by $x_1 = 400, x_2 = 0, x_3 = 200, x_4 = 0$ and $x_5 = 100$. Thus, the answer is that 200 is the largest physically possible value for x_3.

2. Find a 2×2 matrix $X = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}$ that satisfies

$$X + X^t = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix},$$

or explain why no such X exists.

There is no such X.

Solution 1: By equating the left and right hand sides of

$$X + X^t = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix},$$

we obtain the system

$$\begin{array}{lllll} 2x_{11} & & & & = 1 \\ & x_{12} & +x_{21} & & = 2 \\ & x_{12} & +x_{21} & & = 3 \\ & & & 2x_{22} & = 4 \end{array}$$

This system is inconsistent, as one sees by examining the 2nd and 3rd equations.

Solution 2: The matrix on the left, $X + X^t$, is symmetric, while the matrix on the right, $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, is not. Hence there can be no solution.