## Practice about subspaces, bases, dimension

(1) Find bases for Col($A$), Row($A$), and Nul($A$) for

$$A = \begin{bmatrix} 1 & 0 & -1 & 0 & -1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

(2) What are the rank and nullity of the matrix in Problem (1)? Is your answer consistent with the Rank + Nullity Theorem?

(3) The set of vectors $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ satisfying $2x_1 - x_2 + 3x_3 + x_4 = 0$ is a subspace of $\mathbb{R}^4$. Find a basis for this subspace.

(4) Find a subset of

$$S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

that is a basis for the subspace of $\mathbb{R}^4$ spanned by $S$.

(5) True or False: if $A$ is a matrix with RRE form $B$, then the set of pivot rows of $A$ is a basis for Row($A$). (Define a pivot row of $A$ to be a row of $A$ corresponding to a row of $B$ containing a pivot.)

(6) Let $H$ be a subspace of $\mathbb{R}^n$, let $I \subseteq H$ be an independent set, and let $S \subseteq H$ be a spanning set. Show that $|I| \leq |S|$. (Here $|X|$ means "the size of $X$".)