

HISTORY OF MATHEMATICAL IDEAS

MIDTERM

Name: _____

You have 50 minutes for this exam. If you have a question, raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**.

1. Define the following terms.

(a) *commensurable lengths*.

Segments a and b are commensurable if they are “measured by the same standard”, which means that there is a segment c and positive integers m and n such that $|a| = m|c|$ and $|b| = n|c|$.

(b) *algebraic number*.

An algebraic number is a number that is a root of a nonzero rational polynomial.

2. Match the person to the event.

Descartes	Proved that π is transcendental. (Lindemann)
Euclid	Proved a sufficient condition for the constructibility of a regular n -gon. (Gauss)
Euler	Introduced a coordinate system for geometry and characterized the constructible numbers via field extensions. (Descartes)
Gauss	Proved that trisection of a general angle cannot be accomplished with straightedge and compass. (Wantzel)
Lindemann	Founded the school that introduced the word “mathematics”. (Pythagoras)
Plato	Proved that $v - e + f = 2$ for any polygonal dissection of the sphere. (Euler)
Pythagoras	Wrote <i>The Elements</i> . (Euclid)
Wantzel	Associated the regular solids with the classical elements. (Plato)

3. State and prove the Pythagorean Theorem.

See <http://www.cut-the-knot.org/pythagoras/> for many correct answers.

4. Starting with points O and I in the plane, whose distance apart is 1, explain how to construct a segment whose length equals the Golden Ratio.

The Golden Ratio is $\frac{1+\sqrt{5}}{2}$. Hence it can be produced from 1 in four steps:

$$1 \xrightarrow{(a)} 5 \xrightarrow{(b)} \sqrt{5} \xrightarrow{(c)} 1 + \sqrt{5} \xrightarrow{(d)} \frac{1 + \sqrt{5}}{2}.$$

A full solution requires that you explain how to (a) concatenate 5 copies of a given length, (b) form a square root of a given length, (c) concatenate two given lengths, (d) bisect a given segment.