HISTORY (MATH 4820): REVIEW SHEET

I. Math topics.
  (a) Pythagorean theorem (statement and proof).
  (b) Parametrization of conics by rational functions.
  (c) Pythagorean triples.
  (d) Euclidean algorithm.
  (e) Commensurable lengths.
  (f) Rational versus irrational numbers. (Examples of irrational numbers, with proofs.)
  (g) The Golden Ratio.
  (h) Convex polyhedra. (Platonic solids, prisms, antiprisms, Archimedean solids, Johnson solids.)
  (i) The classification of Platonic solids.
  (j) Euler's formula.
  (k) Descartes' Theorem about total defect.
  (l) Euler characteristic.
  (m) Constructibility with straightedge and compass.
    (i) Affine planes are associated to fields. Planes closed under Euclidean constructions are associated to Euclidean fields.
    (ii) The field of constructible numbers is the smallest Euclidean subfield of $\mathbb{R}$.
    (iii) A characterization of the field of constructible numbers via chains of subfields.
    (iv) Constructible numbers are algebraic and have minimal polynomial whose degree is a power of 2.
    (v) Lindemann's Theorem proves that π is not algebraic. Hence circle-squaring is impossible, in general.
    (vi) A monic polynomial with coefficients in the field rational numbers is a minimal polynomial iff it is irreducible.
    (vii) The Rational Root Theorem can be used to test the irreducibility of cubics.
    (viii) $\sqrt[3]{2}$ is not a constructible real number, so cube-doubling is impossible, in general.
    (ix) The polynomial $x^3-3x-2\cos(\alpha)$ is sometimes irreducible over $\mathbb{Q}[\cos(\alpha)]$. Hence angle-trisection is impossible, in general.
    (x) The minimal polynomial of $2\cos(2\pi/n)$ has degree equal to half the number of integers k satisfying $1 \leq k \leq n$ and $\gcd(k,n)=1$. The regular n-gon is constructible iff this number is a power of 2.

II. History topics. (Phrased as questions.)
  (a) What approximate date is assigned to the Ishango bone? Why might this bone be interesting?

(b) What approximate date is assigned to Plimpton 322? What is interesting about this tablet?
(c) Why is Pythagoras remembered?
(d) Why is Hippasus remembered?
(e) Why is Euclid remembered?
(f) Why is Descartes remembered?
(g) Why is De Moivre remembered?
(h) Why is Euler remembered?
(i) Why is Gauss remembered?
(j) Why is Wantzel remembered?
(k) Why is Lindemann remembered? (Hermite?)

General advice on preparing for a math test.
Be prepared to demonstrate understanding in the following ways.

(i) Know the definitions of new concepts, and the meanings of the definitions.
(ii) Know the statements and meanings of the major theorems.
(iii) Know examples/counterexamples. (The purpose of an example is to illustrate the extent of a definition or theorem. The purpose of a counterexample is to indicate the limits of a definition or theorem.)
(iv) Know how to perform the different kinds of calculations discussed in class.
(v) Be prepared to prove elementary statements. (Understanding the proofs done in class is the best preparation for this.)
(vi) Know how to correct mistakes made on old HW.