

History of Mathematical Ideas

Quiz 9

Name: _____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Find the homogeneous form of the polynomial $F(x, y) = y^2 - (x^3 + 1)$.

$$H(X, Y, Z) = Z^3 \cdot F(X/Z, Y/Z) = Y^2Z - (X^3 + Z^3)$$

2. Find the point(s) at infinity in \mathbb{RP}^2 on $y^2 = x^3 + 1$.

We want points on $0 = H(X, Y, Z) = Y^2Z - (X^3 + Z^3)$ which have Z -coordinate zero. Setting $Z = 0$ we obtain $0 = X^3$, which is satisfied only when $X = 0$. Since $X = Z = 0$ at any point at infinity on this curve, and we can scale the coordinates of any triple without changing which point it represents, we can assume that $Y = 1$.

Hence the only point at infinity is $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.