History of Mathematical Ideas
Quiz 6

Name:_____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Give examples of the following types of numbers.
   (a) A number that is the area of a circle that **cannot** be squared with straightedge and compass.

   Area=π (radius $r = 1$). This cannot be squared because some circles of area π are constructible (e.g., the unit circle centered at the origin), while no squares of area π are constructible (since Lindemann's Theorem guarantees that $\sqrt{\pi}$ is not an algebraic number).

   (b) A number that is the area of a circle that **can** be squared with straightedge and compass.

   We can construct a unit square, so any circle of area 1 can be squared.

2. Give examples of the following types of numbers.
   (a) A number that is the volume of a cube that **cannot** be doubled with straightedge and compass.

   Volume = 1. This cannot be doubled because $\sqrt[3]{2}$ is not a constructible real number.

   (b) A number that is the volume of a cube that **can** be doubled with straightedge and compass.

   We can construct a unit cube, so any cube of of volume $\frac{1}{2}$ can be doubled with straightedge and compass.

3. Give examples of the following types of numbers.
   (a) A number that is the measure of an angle that **cannot** be trisected with straightedge and compass.

   Angle $\pi/3$, or $60°$ cannot be trisected, since $2\cos(60°)$ is a constructible real number, but $2\cos(20°)$ is not.

   (b) A number that is the measure of an angle that **can** be trisected with straightedge and compass.

   Easy answer: π is trisectible since $\pi/3$ is constructible. Less obvious answer: $\pi/13$ is not constructible, but it is trisectible.