History of Mathematical Ideas
Quiz 12

Name:_____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Show, or explain, why there exist infinitely many congruent nonoverlapping triangles in the hyperbolic plane. (Hint: Explain how one can start with one triangle and generate infinitely many congruent nonoverlapping copies.)

Start at some point A_0 on the x-axis and mark off other points $A_1, A_2, \ldots$ in the positive x-direction such that $\overline{A_0A_1} = \overline{A_1A_2} = \overline{A_2A_3} = \cdots = 1$. Draw an H-line ℓ_i perpendicular to the x-axis at A_i for each i. Let T_1 be any triangle between ℓ_0 and ℓ_1. Now reflect T_1 through ℓ_1 to create a congruent triangle T_2 that lies between ℓ_1 and ℓ_2. Continue reflecting successively through $\ell_2, \ell_3, \ldots$ and creating congruent reflected images $T_3, T_4, \ldots$ with each T_{i+1} between ℓ_i and ℓ_{i+1}.