

Practice about Pell's equation

Let's find integer solutions to $x^2 - 7y^2 = 1$.

- (1) Find a solution with Bhaskara's method starting with the guess $(1, 1)$. Recall that Bhaskara uses Brahmagupta's Identity:

$$(x_1^2 - Ny_1^2)(x_2^2 - Ny_2^2) = (x_1x_2 + Ny_1y_2)^2 - N(x_1y_2 + x_2y_1)^2$$

- (2) Find a solution using the fact that $\sqrt{7} = [a_0; a_1, a_2, \dots] = [2; \overline{1, 1, 1, 4}]$. (It may help to remember that the convergents p_n/q_n may be generated by the recurrences $p_n = a_n p_{n-1} + p_{n-2}$ and $q_n = a_n q_{n-1} + q_{n-2}$.)

- (3) Generate another solution to $x^2 - 7y^2 = 1$.