

HISTORY OF MATHEMATICAL IDEAS: FINAL EXAM

You have one week to complete this exam. Please turn it in by Wednesday, May 9.

You may use your book, but you may not communicate with others concerning the exam.

In order to receive full credit your answer must be **complete**, **legible** and **correct**.

I have neither given nor received aid on this exam.

Name: _____

1. Find the smallest positive x solving the system

$$\begin{aligned}x &\equiv 3 \pmod{5} \\x &\equiv 3 \pmod{6} \\x &\equiv 5 \pmod{7} \\x &\equiv 5 \pmod{8}\end{aligned}$$

2. This problem concerns the cubic equation $x^3 + 6x - 20 = 0$.

(a) Find all three roots of the equation using Cardano's formula.

(b) Identify the real root (there is only one), and look at a calculator estimate of the real root. Give a proof that the expression from (a) for the real root simplifies to a familiar number.

(c) Give a simple expression for each of the three roots.

3. Show that $\aleph_1 + 1 < \aleph_2$.

4. Prove some theorem using some form of the Cantor diagonalization argument. (Start by stating the theorem you want to prove, then give the proof.)

5. Does there exist a regular tetrahedron that is scissors congruent to the union of two disjoint regular tetrahedra? (Do not assume that any two of these tetrahedra are the same size.)