HISTORY OF MATHEMATICAL IDEAS
MIDTERM

Name:______________________________

You have 50 minutes for this exam. If you have a question, raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**.

1. Define the following terms.

(a) *algebraic number*.

An algebraic number is a complex number that is a root of a nonzero rational polynomial.

(b) *prime number*.

A prime number is an integer $p > 1$ whose only factors are p and 1.

(c) *Euclidean* field. (You may assume that the word "field" is known.)

A Euclidean field is an ordered field closed under square roots of positive elements.

2. Match the person to the event.

Archimedes	Proved that π is transcendental. (Lindemann)
Descartes	Determined which regular n-gons are constructible. (Gauss)
Diophantus	Characterized the constructible numbers via field extensions. (Descartes)
Euclid	Proved that periodic continued fractions represent quadratic irrationals. (Euler)
Euler	Is thought to be the originator of the axiomatic method. (Thales)
Gauss	First written use of the phrase "continued fraction". (Wallis)
Hermite	Wrote the Elements. (Euclid)
Lagrange	Posed the cattle problem, an instance of Pell's equation. (Archimedes)
Lindemann	Founded the school that introduced the word "mathematics". (Pythagoras)
Pythagoras	Proved that quadratic irrationals have eventually periodic continued fractions expansions. (Lagrange)
Thales	Studied integer and rational solutions of polynomial equations. (Diophantus)
Wallis	Proved that e is transcendental. (Hermite)

3. State and prove the Pythagorean Theorem.

See http://www.cut-the-knot.org/pythagoras/ for many correct answers.

4. Find two integral solutions to $x^2 - 5y^2 = 1$ with both x and y positive.

The continued fraction algorithm yields the solution $(9, 4)$.
Then, since $(9 + 4\sqrt{5})^2 = (161 + 72\sqrt{5})$, it follows that $(161, 72)$ is another solution.