History of Mathematical Ideas
Quiz 9

Name:_____

You have 15 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. If $f(x) = x^4 + 6x^2 - 8x + 1$, then $f'(x) = 0$ has exactly one real solution. Find it.

$f'(x) = 4x^3 + 12x - 8$, so we want to solve $4x^3 + 12x - 8 = 0$. This is equivalent to $x^3 + 3x - 2 = 0$.

The roots given by the cubic formula are:

(1) $x = \sqrt[3]{1+\sqrt{2}} + \sqrt[3]{1-\sqrt{2}}$
(2) $x = \omega\sqrt[3]{1+\sqrt{2}} + \omega^2\sqrt[3]{1-\sqrt{2}}$
(3) $x = \omega^2\sqrt[3]{1+\sqrt{2}} + \omega\sqrt[3]{1-\sqrt{2}}$

Since the first root is seen to be real, and we are looking for the unique real root, the answer is $x = \sqrt[3]{1+\sqrt{2}} + \sqrt[3]{1-\sqrt{2}}$.