

History of Mathematical Ideas

Quiz 6

Name: _____

You have 15 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Let a and b be nonzero integers. Show that any integer solution (x_0, y_0) to $aX + bY = \gcd(a, b)$ satisfies $\gcd(x_0, y_0) = 1$.

Let $d = \gcd(a, b)$ and choose a', b' so that $a = da'$ and $b = db'$. Then $aX + bY = \gcd(a, b)$ may be rewritten as $da'X + db'Y = d$, which has the same integer solutions as $a'X + b'Y = 1$. Now, if (x_0, y_0) is a solution of this equation, then (a', b') is a solution of the equation $x_0X + y_0Y = 1$. Since this latter equation is solvable, $\gcd(x_0, y_0)$ must divide 1, hence must equal 1.

2. What are the convergents of $[1; 2, 3, 4]$?

They are:

- $[1] = 1$.
- $[1; 2] = 3/2$.
- $[1; 2, 3] = 10/7$.
- $[1; 2, 3, 4] = 43/30$.