History of Mathematical Ideas
Quiz 10

Name:_____

You have 15 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. What are Viète's Formulas?

If the roots of $x^n - s_1x^{n-1} + s_2x^{n-2} - s_3x^{n-3} + \cdots + (-1)^n s_n = 0$ are $r_1, r_2, \ldots, r_n$, then

$$\begin{aligned}
(s_0 &= 1) \\
s_1 &= r_1 + r_2 + \cdots + r_n \\
s_2 &= r_1r_2 + r_1r_3 + \cdots + r_1r_n + r_2r_3 + \cdots + r_{n-1}r_n \\
s_3 &= r_1r_2r_3 + r_1r_2r_4 + \cdots + r_{n-2}r_{n-1}r_n \\
&\vdots \\
s_n &= r_1r_2\cdots r_n
\end{aligned}$$

2. Use the Newton-Girard Formulas to compute $p_k = r_1^k + r_2^k$ for $k = 1, 2, 3, 4$, where r_1 and r_2 are the roots of $x^2 - ax + b = 0$. (For each k, your answer should be a polynomial expression in a and b.)

We know that $p_0 = 2$, $p_1 = s_1 = a$, and $p_k = ap_{k-1} - bp_{k-2}$, so

$$\begin{aligned}
(p_0 &= 2) \\
p_1 &= \underline{a} \\
p_2 &= a(a) - b(2) = \underline{a^2 - 2b} \\
p_3 &= a(a^2 - 2b) - b(a) = \underline{a^3 - 3ab} \\
p_3 &= a(a^3 - 3ab) - b(a^2 - 2b) = \underline{a^4 - 4a^2b + 2b^2}
\end{aligned}$$