

Theory of Rings Homework

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Rings2p3.1(+converse): Show that if R is semisimple, so is $\mathbb{M}_n(R)$, and show that if $\mathbb{M}_n(R)$ is semisimple, then so is R .

R is semisimple if and only if every object in the category $R\text{-Mod}$ is a semisimple module. But $R\text{-Mod}$ is equivalent to $\mathbb{M}_n(R)\text{-Mod}$. Thus, every object in the category $R\text{-Mod}$ is semisimple if and only if every object in the category $\mathbb{M}_n(R)\text{-Mod}$ is a semisimple module, which is true if and only if $\mathbb{M}_n(R)$ is a semisimple ring.